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**OF YOUNG RESEARCHERS AND SCIENTISTS**

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Tatiana Čorejová
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## SECTION 8

**NATURAL SCIENCES (APPLIED MATHEMATICS)**
**SOCIAL SCIENCES**

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**TRANSCOM 2015**
**11th European conference of young researchers and scientists**

TRANSCOM 2015, the 11th international conference of young European scientists, postgraduate students and their tutors, aims to establish and expand international contacts and co-operation. The main purpose of the conference is to provide young scientists with an encouraging and stimulating environment in which they present results of their research to the scientific community. TRANSCOM has been organised regularly every other year since 1995. Between 160 and 400 young researchers and scientists participate regularly in the event. The conference is organised for postgraduate students and young scientists up to the age of 35 and their tutors. Young workers are expected to present the results they had achieved.

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# CONTENTS

**TRANSCOM 2015, 22-24 June 2015**
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# One-Dimensional Non-Fourier Heat Conduction in the Skin

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**Abstract.** In the paper we deal with a non-Fourier equation describing heat transfer in a flat layer of skin. As it turns out, the skin is an excellent material suitable for the isolation of high temperatures, which has been proven in this work. Compared with other heat conductive materials, for example steel, skin has the best insulation properties.

**Keywords:** Heat pulse propagation, ultrafast heat conduction, Non-Fourier Heat Conduction.

## 1. Introduction

Development of science and technology and the associated need to use different phenomena of thermal conductivity causes still interest in the development of new applications.

A special case is the phenomenon of sudden heat pulse propagation with finite speed. Numerous constitutive connections in continuum mechanics are described on the same idea of a linear association between different features, e.g., the connection between strain tensors in elasticity and stress or rate of strain tensors in a viscous fluid and stress, on the Hook's law. One of the fundamental deficiencies of Fourier's law is that it prompts a parabolic equation for the temperature field. Therefore, it is anticipated that a thermal disorder anytime in a material body will be felt in a split second, however, unevenly, at all different parts of the body. This unreal characteristic is called the "paradox of heat conduction" [1].

In the systems of ultrafast heat conduction, the extent of heat in space or time are greatly short, very high heat fluxes appear and therefore, the classic Fourier conduction law loses its validity. To take such situation into account different mathematical models have been proposed. The primary papers on this subject were shown by Cattaneo and Vernotte, [2]. They define model, which presents the idea of the relaxation time, ***τ*,** as the build-up time for the beginning of the thermal flux after a temperature gradient is suddenly forced on the sample. The relationship between the thermal disorder in the thermal wave model and the heat flux vector is illustrated as [3]:

$$\vec{\mathbf{q}} + \tau_q \frac{\partial \vec{\mathbf{q}}}{\partial t} = -k \nabla \mathrm{T} \tag{1}$$

where $\vec{\mathbf{q}}$ stands for heat flux [W/m$^2$], $\tau_q$ describes relaxation time of the thermal temperature [s], $\mathrm{T}$ is temperature [K], $k$ is thermal conductivity [W/(mK)] and $t$ is time.

For many mediums, e.g. for metals, the relaxation time $\tau_q$ is of the order nanoseconds or picoseconds. In the pulse solid-state laser, the pulse discharge obtains high heat flux density in ultra short time and the discharge plasma is up to high temperature instantaneously ($10^4$ K), giving off radiation of high brightness [4]. However, the thermal relaxation time of the laser medium is much longer or in the same order compared with the pulse duration (for example, $\tau_q \approx 30$ ns, in the event of a typical laser medium Nd:YAG, and that of Nd:GdVo4 is circa 100 ms) [4]. There are several materials where this is not the case, most notably processed meat (15 s), sand (21 s), H acid (25 s), NaHCO3 (29 s), and biological tissue (20–30 s and more) [1].

Here we consider situation that takes place in thin layer of metal and in the skin in the case of puls or short time lasting heating.

## 2. Fourier and Non-Fourier Equations for Heat Conductions

Heat conduction difficulties result from the Fourier's heat-conduction law. This law belies Einstein's relativity concept. The Fourier's law leads to the parabolic equation of heat conduction

$$\nabla^2 T - \frac{1}{a}\frac{\partial T}{\partial t} = -\frac{Q}{k}$$

where $a=k/\rho c$ is the thermal diffusivity [m$^2$/s], $\rho$ is density of the material [W/m$^2$], $c$ denotes specific heat [J/(kgK)], $Q$ stands for source of heat [W/m$^3$].

Fourier's law expects that any thermal disruption on a body proliferates at an infinite speed. Practically, thermal wave speed is very high. For metals it can be of the order $10^3$ [m/s]. Here we consider the heat-conduction law described by (1), i.e. we take into account situation that velocity of thermal signal is finite. Hence, we obtain hyperbolic equation of heat conduction (without sources of heat) [5]:

$$\rho c\left(\frac{\partial T}{\partial t} + \tau_q \frac{\partial^2 T}{\partial t^2}\right) = \nabla \cdot (k \nabla T). \tag{2}$$

For one-dimensional case and $k$ = const we can formulate the equation (3) in the form

$$\tau_q \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \quad . \tag{3}$$

For human skin we accept the following thermophysical properties: thermal conductivity $k = 0.235$ [W/(mK)], $\rho c = 4.284 \cdot 10^6$ [J/(m$^3$K)], $\tau_q$ is circa 10 [s], where the thermal wave velocity in the skin $v = \sqrt{\frac{k}{\rho c \tau_q}} = \sqrt{\frac{a}{\tau_q}}$, is equal approximately $7{,}4 \cdot 10^{-5}$ [m/s].

Consider a problem of heating a layer of skin of thickness $L = 0.02$ m. The initial and boundary conditions we accept in the form:

$$T(x,0) = 0, \qquad \frac{\partial T}{\partial t}(x,0) = 0, \tag{4}$$

$$T(0,t) = T_0 = 100\ ^{o}C, \qquad T(L,t) = 0. \tag{5}$$

The exact solution has been derived based or results of papers [6] and [7]. Solution of the problem (3), (4), (5) in dimensionless coordinates ($\tau = t/(2\tau_q)$, $x = x_{\mathrm{real}}/L$) reads

$$T(x,\tau) = T_0(1-x) - 2T_0 e^{-\tau}\sum_{n=1}^{N}\frac{\sin(\pi n x)}{\pi n}\left[\frac{\sinh(q_n\tau)}{q_n} + \cosh(q_n\tau)\right] + \\ - 2T_0 e^{-t}\sum_{n=N+1}^{\infty}\frac{\sin(\pi n x)}{\pi n}\left[\frac{\sin(\mathrm{p}_n\tau)}{\mathrm{p}_n} + \cos(\mathrm{p}_n\tau)\right]$$

with

$$q_n = \sqrt{1 - \frac{4a\tau_q}{L^2}\pi^2 n^2}\ , \qquad p_n = \sqrt{\frac{4a\tau_q}{L^2}\pi^2 n^2 - 1}$$

and $N = \left[\sqrt{\frac{L^2}{4a\tau_q\pi^2}}\right]$ is the integer part of $\sqrt{\frac{L^2}{4a\tau_q\pi^2}}$ .

It is clear that the thermal signal reaches points inside the skin with a delay. For x = 0.005 the delay is of the order of 0.1 (in seconds time *t* equals 0.2 sec); for x = 0.05 we get *t* = approx. 14 sec, for x = 0.25, we have *t* = approx. 67,6 sec, for x = 0.5 we find *t* = approx. 135,1 sec and for x = 1 the time *t* is already counted in minutes (approx. 4.6 minutes). Moreover, since the boundaries are held constant temperatures, $T_0$ and zero, respectively, the temperature distribution in the skin will tend to a linear decreasing from $T_0$ to zero. For instance, for x = 0.005 and dimensionless time equal 0.1 the temperature is equal circa 93°C while for x = 0.05 and dimensionless time 0.85 the temperature is equal 53.57°C. For this reason, the temperature achieved in the subsequent points of the skin will become smaller and smaller. This is illustrated in Figure 1.

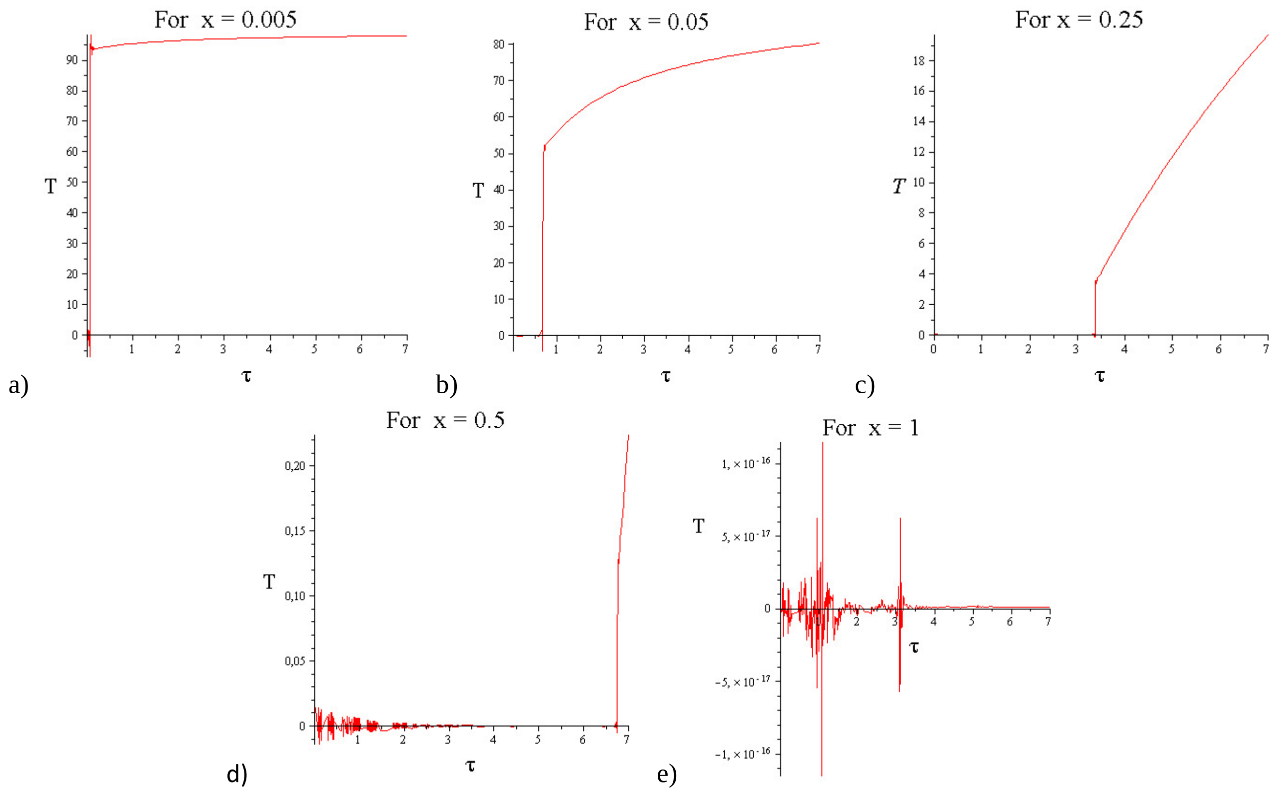


**Fig. 1.** Temperature as a function of dimensionless time τ obtained for dimensionless thickness a) x = 0.005, b) x = 0.05, c) x = 0.25, d) x = 0.5, e) x = 1

The temperature distribution for dimensionless time τ = 0.7, 1.4, 2.0 (dimensional time = 14, 28 and 40 sec) is presented in Figure 2. Time after which the temperature signal reaches the other side of the skin (*L* = 0,02 cm) is dimensionless 270.2 and 540.4 seconds dimensionally.

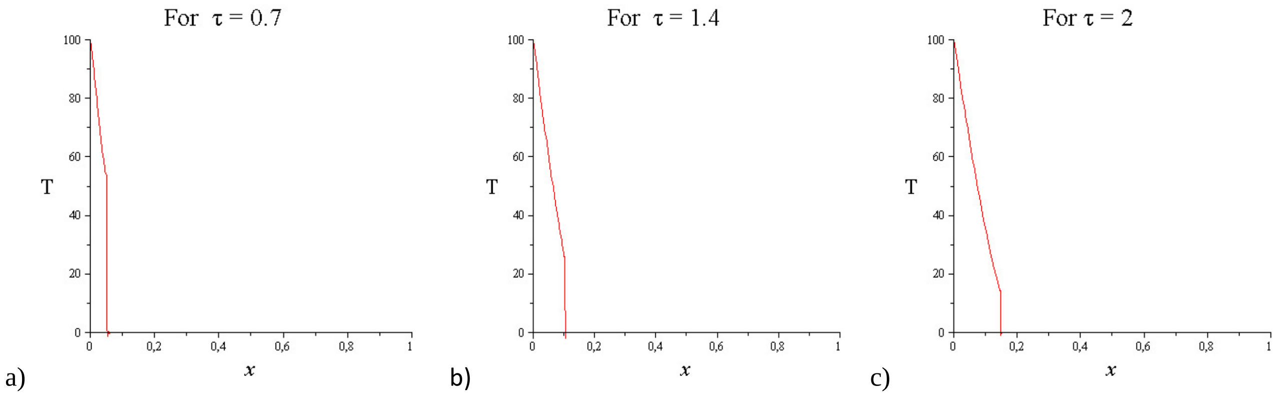


**Fig. 2** Temperature obtained for dimensionless time a) τ = 0.7, b) τ = 1.4, c) τ = 2

In Figure 2 we see that when the time increases, the temperature drops slower along x axis, for example for $\tau = 0.7$ and $x = 0.05$ the temperature $T = 50.69^{\circ}C$, and for the same value of x and for $\tau = 1.4$ the temperature is $T = 60.44\ ^{\circ}C$.

For skin material with a thickness x = 0.02 m real time of the thermal signal transition to the other side of the material is equal to 540.4 seconds. For comparison, the thermal signal transition time to the other side of the steel material with the same thickness is $8.93 \cdot 10^{-5}$ seconds (for steel with relaxation time, supposably, $\tau_q \approx 10^{-10}$ s and thermal diffusivity $a \approx 5 \cdot 10^{-6}\ m^2/s$ the speed of the thermal wave $c \approx 224$ m/s [1]). As a conclusion we can state that the construction of insulating barrier, whose job is to protect against sudden but short heat pulse, one should use a thick layer of the skin as an insulation material.

## 3. Conclusion

The skin has an excellent property of passive heat conduction. In response to the very high ambient temperature the skin conducts heat very slowly into its depths.

Assuming that the skin thickness is 2 cm, the thermal signal penetrates the skin surface in real time t = approx. 570 [s]. Comparing skin with the steel is easy to notice that the time interval of thermal signal transition in the steel of the same thickness is approximately $6 \cdot 10^5$ shorter.

The skin makes very slow transition of thermal signal through the barrier. Therefore skin seems to be a very good material suitable for the construction of insulating bulkheads protecting against sudden but short heat pulse.

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# Traditional and Urban Legends of Today's Information-Communication Society

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**Abstract.** The presented contribution focuses on concept of the traditional and urban legends, that are despite developed society less known and are not given proper attention. But Legend still has everyday important place in the communication and in the virtual environment. The main aim of research is to create the concept and public space for preservation, documentation, digitalisation and access to traditional and contemporary legends to public.

**Keywords:** Traditional and urban legend, narratives, cultural heritage, communication, presentation, information and communication society.

## 1. Introduction

Our ancestors were forwarding their heritage and precious knowledge by different methods from generation to generation. This forwarding was mediated by wall paintings, smoke signals, storytelling and later by recording by the written text. The invention of printing was a big turning point for human civilization. Many stories preserved in book form were acquired by people and broadened them as their own experiences, and thus enlarged their narrative repertoire.

Rumors reveal the identity of the society and describe the cultural values that are important in real or virtual environment. We can apprehend the essence of legends keeping and literature in two ways, both in terms of past and future. In view of the past, it is obtaining and keeping of existing stories, rumors that have been reported, either through personal written documents, recordings, general chronicles and archives or in the memory of our ancestors, the older generation and are linked to a specific period of time of their everyday life.

They help us to create an image of the past and the history. In view of the present and the future there are the rumors that arise in the current society full of info-communication technologies. Legends have survived, they are spreading and constantly forming in the form of urban legends that grow most in the Internet environment, which offers fast, global and cheap spreading of rumors in real time.

Unfortunately, there are many stories which emerges in this environment and brings people to confusing and stressful situations. They are called hoaxes, chain messages that spread quick as snow avalanche. They are so convincing that the general public does not know whether they are true or not. And this is precisely their role.

There are various types of hoaxes:

- description of unreal danger, various warnings and rumors,
- warnings against viruses and various attacks on your computer,
- hoaxes related to social networks,
- false requests or old requests for help,
- rumors about mobile phones,
- petitions and appeals,
- pyramid games and various offers for easy money,
- chain letters and funny messages,

- specific forms of hoax (scamming, phishing, false lottery),
- urban legends.

Current legends presents an inseparable part of Slovakian folklore of today. They are too good to be true. They are stories from everyday life that happened to a friend of a friend. The current legends are mostly untrue, meaning they could not have happened to hundreds of friends of friends, family members, cousins, hairdressers, drivers, taxi drivers, colleagues and to whoever they are addressed. Surely many of you know stories about infected injections in public transport vehicles or razors on pool slides. As a typical example of funny and absurd situation we can use a story about young teenagers who were drunk and made a bet that none of them will put a lightbulb in their mouth. One of them managed to do it, but he couldn't take it out. After trying for a long time, they decided to take their unfortunate friend to emergency. It was late night hour and none of them could drive, so they called a cab. The driver looked at the young man and asked what happened. When told, he laughed a lot. At the emergency they took the lightbulb out, so the young men could go home. When they returned home, they saw another one of their friends trying to take a lightbulb out of his mouth. So they had to return to the emergency room. In the waiting room there was a familiar face of the taxi driver with a lightbulb in his mouth.

Current legends are changing constantly. Every storyteller edits his story according to the circumstances, listeners and environment. For this reason it would be good for our society to store this kind of stories, too.

However in the presented article the main things of interest are traditional legends that lose their character and function in the environment of information-communication society.

Based on the finding above about the importance of storing traditional legends in current society we decided to make a detailed analysis of literature of Kysuce area, its division into villages and towns and the most frequently repeated demonological characters in stories typical for these areas. This analysis will be specified in the following part. However at first it is important to specify the term legends and their classification.

## 2. Classification of Legends

Legends are linked in particular to the place of their origin and they gain its local character. Their content follows the events, characters and memories which were debated at home or at the nearest region. The main mediator is a storyteller who has adapted the story to his environment or given region, which he knew intimately since his/her childhood. [1]

Our focus will be on superstitious legends, respectively stories (stories between a fairy tale and legend). There is a brief narration about experiences and meetings with supernatural beings, unexplained phenomena and appearances at unusual places and so on. Such supernatural experiences are often described in several legends at Kysuce region. These legends will be approximated in the following analysis of research published samples of books and articles of Kysuce legends and superstitious creatures typical of this region. These stories are referred as demonological legends, which are distinguished by superstitious beings and phenomena, which are divided into the following groups: [2]

First group is represented by revenants, the souls of the dead, which did not have a rest from several reasons after death and their souls were coming back among the living (non-baptised crying baby, that is still heard near the swamps, ...).

Second group represents a real, living people who are usually called witches or sorcerers, who based of their magic caused the diseases to humans, animals, and even death. According to their skills, we distinguish three categories of individuals:

- persons who do not have supernatural abilities, but using media containing harmful magical powers,
- people who had supernatural powers, especially witches that have ability to transform into different animals (black cat, frog, dog, crow or various creatures of evil: hag, vampire, ...)

- persons treated as witches, wizards, enchanters, which were said to acquired their magic powers of the devil itself.

Third group creates a demonological characters of animistic (soul) and manic origin such as various animals (crayfish, snake, ...), but also fairies, water sprites and sorcerers. Very distinctive character is the devil, representing a harmfull demon.

Based on the results obtained during the analysis of literature and yet published contributions in the press of regional newspapers is more than clear that from region Horné Kysuce (Upper or North Kysuce), namely village *Turzovka*, was paid more attention than other parts of the region (Upper, Lower, Middle and Eastern Kysuce). But even though the number of existing legends, involve the individual municipalities is really minimal.

The most common demonological characters and objects that are mentioned in Kysuce legends are[1]:

- **Ghosts:** mostly represent lost souls who cannot find rest in the thereafter. Were killed, unburied by Christian habits and constantly wander through the world of the living.
- **Fairies**: female creatures that may be water, mountain and forest based. We classify to them also the Fates that puts good or bad qualities to newborn´s dowry.
- **Witches**: women, which certainly does not resemble attractive models. They try to harm people, but is possible to protect ourselves by using different magic potions and special formulas.
- **Torchbearers**: were very popular figures, which were at Kysuce tales often minded. We have not found in the dictionary the definition of this famous character, but its further description is presented below.
- **Water spirit:** soul of the water realm. He takes care of souls of drowned people, which were very often drowned by him. He was able to transform into various animals, whether huge crawfish, frog, fish or white or a black dog. He needed only a drop of water to do this. His place of activity was most mainly near marshes, streams and ponds. He was often seen near the Watermills.
- **Devil:** The devil, citizen of Hell, who deceives the people astray. He is described mostly with small horns on top of the head, sulfur odor and with hoof at one of the legs. He have a human form look, which he changes depending on who wants to get into hell. Either it is a young man, sometimes the hunter with typical green vest with a red hat and feather. He is able to transform into animal form also.
- **Leprechaun/goblin:** ghost, whose main characteristic is treacherousness. Also take various appearances, such as black or red fiery chicken rooster.
- **Jánošík and bandits.**
- **Burried, cursed and hidden treasures.**

## 3. Legends from Kysuce

Legends from Kysuce region usually speaks about the emergence of cultural and historical phenomena such as municipalities, hamlets, rocks, settlements, dangerous places that everyone bypasses (wetlands, etc.). Furthermore about the actions of prominent heroes who were not afraid of anything and defended not only their families but also the whole village. They explain the various events which affected the destiny of the wider society, as demonstrated by the first and second world war, SNP (Slovak National Uprising), disasters (fires and floods). They most speaks about a better future for the people or an individual by finding some hidden treasure. Directions to treasure have been shown by a water sprite, who wanted to exchanged it for something (child, woman,

[1] Malý slovník strašidiel: SZÁBO, I., STANO, M. Slovenské strašidlá od „a“ po „ž“. Legendy, povesti, príbehy. Bratislava, 2003. 152 s. ISBN 80-85451-13-x.

service), but he has been always deceived by a wise farmer which passed through his mind skilfully. Demonological characters were seen and revealing in their natural environment, from which all the legends arose. Very common were talking about meetings with the dead or about death itself. People were recognizing also several signs of death, such as fallen paintings, blowned out burning candle, revelation of unusual bird. One of the stories is about a woman who died and left a small child. Therefore, she was returning every night to calm her baby down. There are a lot of stories connected according to these legends in Kysuce region, mainly about Torchbearer who were representing the souls of unbaptized children, unveiling people out of their way to get lost. You could not whistle on a Torchbearer, because he would pee on you. Respondents in personal meetings with the story author were minding back that if the someone fled the Torchbearer, he peed his whole house. Other favorite characters were water sprites who pulled people into streams and lakes. Drunken guys who were returning home from the pub had to fight with him not to shove them from the bridge into the river. Water sprite knew to transform into different forms like huge lobster, black dog or a cat. Similarly, also demons and evil spirits generally tried to harm ordinary farmers and citizen.

Other frequently repeated stories were about treasures, with dwarfs or snake appearing near by. Furthermore, it were places, mainly dangerous for young men or youngsters where they could meet with fairies. Fairies danced with the men until you guys overturned their last piece of clothing. If so, fairies disapperaed, if not, they danced with the young men till their end.

In some cases, it was the cock that crowed in the morning and that finished the deadly dancing and screaming of these fairies. Very common element in these stories was, that people with supernatural abilities and characteristics were minded back. They were especially witches who hurt animals (cows were losing milk, ...), destroyed the crops and harvest. Others could bewitch a person. Witches were able to transform their shape into animals, especially frogs and of course a black cats. They were walking through the meadows at St. John´s celebration, and were harming cattles. People could defend against witches by rolling the egg on the back of a cow. Now we can also include talks about nightmares, and Sotona´s, which were choking them at night people and they could not left them to sleep.

Many minded demonological legends may be incomprehensible for a modern man, but in the past, our ancestors believed in these stories. They were closely associated with folk beliefs with different fantasies, which were extended up to historical times. Even today, we are dealing with various unexplained phenomena. Old superstitious stories were talking about similar phenomena, but it was used a different terminology to define all supernatural.

## 4. Methods Suitable for the Storage of Legends and Tales

Nowadays there is no space mapping, storing, documenting, digitalization and sharing of traditional and current legends of the Slovak republic. For this reason we suggest two ways and methods of storing the legends. Creating a website www.povestislovenska.sk which would include available theoretical knowledge about traditional and current legends, results of analysis of online library catalogues stored on the website, organized for individual areas in alphabetical order. There would also be a database of traditional and current legends, free space for site users and its creators as well.

This would create a network of volunteers – whether experts, scientists or lovers of cultural heritage of Slovakia, also a space for documentation and digitalized sources (publications, chronicles, archives) containing legends, while keeping the respective copyrights. There would also be records of so far unknown stories, available in written, acoustic or graphic form.

The other possibility would be creating a book of not completely mapped area of Kysuce, which would serve as a sample of complete research of so far unmapped areas of the Slovak republic.

## 5. Conclusion

Legends are not only a testimony about the history of our country, but also a overall look at the way of life and daily work of our ancestors. Legend, as a part of the non-material cultural heritage has become many times an object of interest and research. However, most of the obtained results are not accessible and affordable for the public. Luckily, with the expansion of new and advanced technologies into the daily lives of ordinary people it was added a unique space for sharing and preservation of traditional and contemporary legends - the Internet.

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# Statistical Inference in Narrow Sense and Approximation of Degrees of Freedom

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**Abstract.** In this paper we derive explicit expression how to approximate the degrees of freedom in case of statistical inference in narrow sense. The presented theoretical approach is based on one linear combination of the fixed and random effects, the Laird-Ware linear mixed model, the Henderson's mixed model equations, the Wald-type statistic and the REML estimates of covariance components. The derived expression can be implemented in any software which has not been able to make statistical inference in narrow sense yet.

**Keywords:** covariance parameters, degrees of freedom, Laird-Ware linear mixed model, matrix differentiation, Wald-type statistic.

## 1. Introduction

The linear mixed models containing the fixed and random effects are useful tool for statistical data analysis, because they enable research workers to make two types of statistical inference simultaneously: broad inference and narrow inference. Both of them are discussed in [5], [7] and [13]. Broad inference represented by linear combination of the fixed effects is applicable to the entire population from which a random sample is chosen. Statistical inference in narrow sense represented by linear combination of the fixed and random effects narrow the area of inference from the entire population only to just those levels determined by linear combination of the random effects.

The longitudinal or repeated measures data are common in many research studies and the Laird-Ware model is the most frequently used statistical tool for these data sets.

## 2. Laird-Ware Model

The Laird-Ware model defined in [4] is one of the special types of the linear mixed models and it has the following form:

$$y_i = X_i\beta + Z_i u_i + e_i, \quad \text{for} \quad i = 1, \dots, s, \tag{1}$$

where $y_i$ is the $n_i$-dimensional column vector of observations on the $i$-th unit, $X_i$ and $Z_i$ represent the known $(n_i \times p)$ and $(n_i \times k)$ design matrices, respectively. The $p$-dimensional column vector $\beta$ and $k$-dimensional column vector $u_i$ represent unknown the fixed and random effects, respectively, and $e_i$ is the $n_i$-dimensional column vector of random errors, for $i = 1, \dots, s$. The random variables $u_i$ and $e_i$ are normally distributed with mean $0$ and $(k \times k)$ and $(n_i \times n_i)$ symmetric covariance matrices $G_0(\theta_1)$ and $R_i(\theta_2)$, respectively. In general, these matrices depending on some vectors of usually unknown parameters $\theta_1$ and $\theta_2$ can have various form, see [9] for more details.

In this paper we restrict to the mixed model with two random effects represented for example by random intercept and random slope. This is common in many situations, for more details see [6].

In our case the matrix $Z_i$ and the covariance matrix $G_0(\theta_1)$ have dimensions $(n_i \times 2)$ and $(2 \times 2)$, respectively. We consider that $G_0(\theta_1)$ is non-singular and has unstructured covariance structure in the following form:

$$G_0(\theta_1) = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \tag{2}$$

which means the matrix $G_0(\theta_1)$ depends on three covariance parameters $\theta_1 = (\sigma_{11}, \sigma_{21}, \sigma_{22})$. Further, we assume the special type of the covariance structure called variance components for matrix $R_i(\theta_2)$, it depends on one parameter $\theta_2=(\sigma^2)$, i.e. $R_i(\theta_2)=\sigma^2 I_{ni\times ni}$ with $(n_i\times n_i)$ identity matrix $I$. We will hereafter use only symbols $G_0$ and $R_i$ for notational simplicity in the text below.

Because of simplification we connect the items from (1) and we put new marks in the following way: $y = (y_1', y_2', \ldots, y_s')'$, $X = (X_1', X_2', \ldots, X_s')'$, $Z = diag(Z_1, Z_2, \ldots, Z_s)$, $u = (u_1', u_2', \ldots, u_s')'$, $e = (e_1', e_2', \ldots, e_s')'$, $\sum_{i=1}^{s} n_i = n$. In conformity with our new symbols the vector of all observations $y$ is normally distributed with mean $X\beta$ and covariance matrix $V = ZGZ' + R$ depending on four covariance parameters $\theta=(\sigma_{11}, \sigma_{21}, \sigma_{22}, \sigma^2)$, where $V = diag(V_1, V_2, \ldots, V_s)$, $V_i = Z_i G_0 Z_i' + R_i$, $G = I_{s\times s} \otimes G_0$ and $R = diag(R_1, R_2, \ldots, R_s)$. We assume the matrix $X$ has the full column rank $r_x$.

To make statistical inference in narrow sense, in our consideration we restrict only to one predictable function (i.e. one linear combination of the fixed and random effects)

$$w = k'\beta + l'u = \lambda' \begin{pmatrix} \beta \\ u \end{pmatrix} \tag{3}$$

provided $k'\beta$ satisfies the estimability requirement and $\lambda' = (k' \;\; l')$.

The approach to statistical inference in narrow sense presented in this paper is based on

- The restricted maximum likelihood function (*REML*) developed by [8] which logarithm is

$$REML: \quad l(\theta \mid y) = -\frac{n-r_X}{2}\ln(2\pi) - \frac{1}{2}\ln\left|M'VM\right| - \frac{1}{2}y'M\left(M'VM\right)^{-1}M'y \tag{4}$$

where $M' = C(I-XX^+)$, $C$ is any matrix with the full row rank $n$-$r_x$ and $X^+$ is Moore-Penrose inverse of $X$. The covariance parameter estimates (the covariance parameters are usually unknown and they must be estimated) can be obtained by equating to zero the first-order partial derivatives of function (4) and software like SAS or R provide these estimates.

- The mixed model equations developed by Henderson in [3]

$$\begin{pmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z + G^{-1} \end{pmatrix} \begin{pmatrix} \beta \\ u \end{pmatrix} = \begin{pmatrix} X'R^{-1}y \\ Z'R^{-1}y \end{pmatrix} \tag{5}$$

which provide the estimates of $\beta$ and predictors of $u$ working on the assumption that covariance parameter estimates are available. We denote the left-hand side matrix in (5) as $H$.

- The Wald-type statistic in the form

$$t = \frac{\hat{w} - w}{\sqrt{\lambda' H^{-1} \lambda}} \tag{6}$$

which is approximated by Student′s distribution with unknown degrees of freedom $v$ and $\hat{w}$ is predictable function with $\hat{\beta}$ and $\hat{u}$ based on covariance parameter estimates.

The problem how to estimate the degrees of freedom for Student's distributed Wald-type statistic on the above mentioned assumptions is solved in next section.

## 3. Method for Degrees of Freedom Approximation

To solve the problem mentioned above, we follow in ideas by Witkovský in [12] and we use similar Satterthwaite's approximation [10] like Giesbrecht and Burns in [1] who approximated statistic for inference in broad sense. We get the degrees of freedom estimate by the following formula:

$$\hat{v} = \frac{2\left(\lambda'\hat{H}^{-1}\lambda\right)^2}{\mathrm{var}\left(\lambda'\hat{H}^{-1}\lambda\right)} \equiv \frac{2\left(\lambda'\hat{H}^{-1}\lambda\right)^2}{\hat{g}'_\lambda \hat{\Sigma}\hat{g}_\lambda} \tag{7}$$

where $\hat{g}_\lambda$, $\hat{\Sigma}$, $\hat{H}$ are $g_\lambda$, $\Sigma$, $H$ evaluated at the covariance parameter estimates, $g_\lambda$ is gradient of $\lambda' H^{-1}\lambda$ and $\Sigma$ is asymptotic covariance matrix of the covariance parameters.

Now we derive explicit expressions for $g_\lambda$ and $\Sigma$ to enable future implementation of this method in some software. In our case

$$g_\lambda = \left(\frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{11}}, \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{21}}, \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{22}}, \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma^2}\right)' \tag{8}$$

and using the formulas for matrix differentiation mentioned in [2] we get the general formula for the first three elements of the vector $g_\lambda$ in the following form

$$\frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{ij}} = -\lambda' H^{-1}\begin{pmatrix} 0_{p\times p} & 0_{p\times ks} \\ 0_{ks\times p} & -diag(B_{ij}, B_{ij}, \dots, B_{ij}) \end{pmatrix} H^{-1}\lambda \tag{9}$$

where the form of $B_{ij}$ in (9) depends on the covariance parameters $\sigma_{ij}$ and these forms are given in (10) – (12):

$$B_{11} = \frac{1}{\left(\sigma_{11}\sigma_{22} - \sigma_{21}^2\right)^2}\begin{pmatrix} \sigma_{22}^2 & -\sigma_{21}\sigma_{22} \\ -\sigma_{21}\sigma_{22} & \sigma_{21}^2 \end{pmatrix} \quad \text{for} \quad \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{11}} \tag{10}$$

$$B_{21} = \frac{1}{\left(\sigma_{11}\sigma_{22} - \sigma_{21}^2\right)^2}\begin{pmatrix} -2\sigma_{21}\sigma_{22} & \sigma_{21}^2 + \sigma_{11}\sigma_{22} \\ \sigma_{21}^2 + \sigma_{11}\sigma_{22} & -2\sigma_{11}\sigma_{21} \end{pmatrix} \quad \text{for} \quad \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{21}} \tag{11}$$

$$B_{22} = \frac{1}{\left(\sigma_{11}\sigma_{22} - \sigma_{21}^2\right)^2}\begin{pmatrix} \sigma_{21}^2 & -\sigma_{11}\sigma_{21} \\ -\sigma_{11}\sigma_{21} & \sigma_{11}^2 \end{pmatrix} \quad \text{for} \quad \frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma_{22}} \tag{12}$$

We get the last element of the vector $g_\lambda$ by similar way and it has the following form:

$$\frac{\partial\left(\lambda' H^{-1}\lambda\right)}{\partial\sigma^2} = \frac{1}{\left(\sigma^2\right)^2}\lambda' H^{-1}\begin{pmatrix} X'X & X'Z \\ Z'X & Z'Z \end{pmatrix} H^{-1}\lambda \tag{13}$$

and we remark that the element in (13) has the same form like in [12], because the same covariance structure of the matrix *R* is taken into account.

The elements of asymptotic covariance matrix of the covariance parameters are based on the inverse of second-order partial derivatives of *REML* function (4) with regard to its parameters

$$\Sigma = 2\cdot\begin{pmatrix} \frac{\partial^2 l}{\partial\sigma_{11}^2} & \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma_{21}} & \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma_{22}} & \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma^2} \\ \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma_{21}} & \frac{\partial^2 l}{\partial\sigma_{21}^2} & \frac{\partial^2 l}{\partial\sigma_{21}\partial\sigma_{22}} & \frac{\partial^2 l}{\partial\sigma_{21}\partial\sigma^2} \\ \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma_{22}} & \frac{\partial^2 l}{\partial\sigma_{21}\partial\sigma_{22}} & \frac{\partial^2 l}{\partial\sigma_{22}^2} & \frac{\partial^2 l}{\partial\sigma_{22}\partial\sigma^2} \\ \frac{\partial^2 l}{\partial\sigma_{11}\partial\sigma^2} & \frac{\partial^2 l}{\partial\sigma_{21}\partial\sigma^2} & \frac{\partial^2 l}{\partial\sigma_{22}\partial\sigma^2} & \frac{\partial^2 l}{\partial\sigma^2\partial\sigma^2} \end{pmatrix}^{-1} \tag{14}$$

Before we derive the form of right-hand side matrix elements in (14) we put these notations

$$G_0^{11} = \frac{\partial G_0}{\partial \sigma_{11}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad G_0^{21} = \frac{\partial G_0}{\partial \sigma_{21}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad G_0^{22} = \frac{\partial G_0}{\partial \sigma_{22}} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad (15)$$

$$P_{ij} = M'Z \frac{\partial G}{\partial \sigma_{ij}} Z'M , \quad \text{where} \quad \frac{\partial G}{\partial \sigma_{ij}} = I_{s \times s} \otimes G_0^{ij}, \quad i,\ j = 1, 2 \qquad (16)$$

Using the formulas for matrix differentiation as above and notations from (16) we get for the matrix elements in (14) these following general formulas

$$\begin{aligned} \frac{\partial^2 l}{\partial \sigma_{ij} \partial \sigma_{kl}} = & \frac{1}{2} tr\left[ (M'VM)^{-1} P_{kl} (M'VM)^{-1} P_{ij} \right] - \\ & - \frac{1}{2} y'M (M'VM)^{-1} P_{kl} (M'VM)^{-1} P_{ij} (M'VM)^{-1} M'y - \\ & - \frac{1}{2} y'M (M'VM)^{-1} P_{ij} (M'VM)^{-1} P_{kl} (M'VM)^{-1} M'y \end{aligned} \qquad (17)$$

$$\begin{aligned} \frac{\partial^2 l}{\partial \sigma_{ij} \partial \sigma^2} = & \frac{1}{2} tr\left[ (M'VM)^{-1} M'M (M'VM)^{-1} P_{ij} \right] - \\ & - \frac{1}{2} y'M (M'VM)^{-1} M'M (M'VM)^{-1} P_{ij} (M'VM)^{-1} M'y - \\ & - \frac{1}{2} y'M (M'VM)^{-1} P_{ij} (M'VM)^{-1} M'M (M'VM)^{-1} M'y \end{aligned} \qquad (18)$$

$$\begin{aligned} \frac{\partial^2 l}{\partial \sigma^2 \partial \sigma^2} = & \frac{1}{2} tr\left[ (M'VM)^{-1} M'M (M'VM)^{-1} M'M \right] - \\ & - y'M (M'VM)^{-1} M'M (M'VM)^{-1} M'M (M'VM)^{-1} M'y \end{aligned} \qquad (19)$$

for $i, j, k, l = 1,2$ and *tr* is the abbreviation of the trace of matrix.

## 4. Conclusion

We worked on the assumptions that only one predictable function is considered and the unstructured covariance matrix $G_0$ has the smallest permissible order. The explicit expression for degrees of freedom estimating in case of the statistical inference in narrow sense can be implemented in some software. The above mentioned problem for more predictable functions and with unstructured covariance matrix $G_0$ of the higher order requires additional considerations.

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# Stability Analysis of Demand-Inventory Model in a Certain Business Case

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**Abstract.** Stability analysis of demand-inventory model formulated with a three-dimensional discrete dynamical system is presented in this paper. The model originates in the paper [3] of Ma-Feng and a few modifications have been implemented. Some of parameters' values have been fixed arbitrary in order to reflect real business case, which namely is a sell-out of a product with a discount dependent on an overstock value. Hence, the analysis is performed for a one-parameter family of maps.

**Keywords:** demand, inventory, discrete dynamical system, difference equations, stability, bifurcation.

## 1. Introduction

In the article [3] of Ma-Feng the dynamical model for demand and inventory has been proposed and its stability using numerical methods has been analyzed. The model describes the process of stock supply of one good (product) at one echelon of a supply chain - at retailer. The authors has been analyzed stability of the proposed dynamical system: using numerically prepared bifurcation diagrams they gave parameters' ranges for stability and chaotic behavior. The authors of this article showed in [2] possible applications of the model in real business cases. The model describes the supply chain consisting of customers, retailer and manufacturer.

### 1.1. The Original Model of Ma-Feng

In the original model of Ma-Feng following rules have been applied:

- customers buy a good from a retailer,
- customer demand depends on a retail price,
- the retailer orders a good according to the forecast dependent on sales and forecast in a previous period,
- a manufacturer produces and delivers the exact ordered amount and production capacity is unlimited,
- the retailer can offer a discount depending on stock volume: when stock is high, the retailer offers a discount to encourage customers to make a purchase.

The model is then set as a system of three difference equations with three variables:

$$\begin{cases} D_{t+1} &=& \dfrac{(qaT)^k \Delta p_{t+1}^{-2c}}{[(qa+1)T - S_{t+1}]^k} D_t \\ S_{t+1} &=& \check{D}_{t+1} + S_t - D_t \\ \check{D}_{t+1} &=& \alpha D_t + (1-\alpha)\check{D}_t, \end{cases} \qquad (1)$$

where:

| | |
|---|---|
| $t = 0, 1, 2, ...$ | indicates time instances, |
| $S_t$ | is a stock volume at *t*, |
| $D_t$ | is a demand volume at *t*, |
| $\check{D}_t$ | is a forecast of demand at *t* and ordered placed by a retailer at a manufacturer, moreover by assumption of unlimited capacity it is also delivered quantity at *t*, |
| $\Delta p_{t+1}$ | indicates change of the price from time *t* to *t*+1, |
| $q,\ a$ | are parameters for discount steering, $q>0$, $a>0$, |
| $T > 0$ | is a parameter for defining the target stock, |
| $c,\ k$ | are price elasticity coefficients, $c<0$ is related to a regular price and $k>0$ to a discount, |
| $0 \leqslant \alpha \leqslant 1$ | is a forecast smoothing coefficient. |

### 1.2. Modified Model

In regard to the original model let us assume that:

- price cannot be arbitrary changed, then $(\Delta p_{t+1})^{-2c} = 1$, for all *t*,
- $S_0 \geq T$,
- $D_{t+1}$ depends on an inventory level at *t*, and not at *t*+1, i.e. $S_{t+1}$,
- $S_{t+1}$ depends on values at *t*, which in practical sense may mean that is measured at the beginning of a day,

and define $A = aq > 0$ without loss of generality, since both parameters always appear together. Hence, the model takes a form:

$$\begin{aligned} D_{t+1} &= \left[\frac{AT}{(A+1)T - S_t}\right]^k D_t \\ S_{t+1} &= S_t - D_t + \check{D}_t \\ \check{D}_{t+1} &= \alpha D_t + (1-\alpha)\check{D}_t \end{aligned} \quad . \tag{2}$$

Let us represent it as a mapping of the form:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} f_1(x_t, y_t, z_t) \\ f_2(x_t, y_t, z_t) \\ f_3(x_t, y_t, z_t) \end{pmatrix} = \begin{pmatrix} \left[\frac{AT}{(A+1)T - y_t}\right]^k x_t \\ y_t - x_t + z_t \\ \alpha x_t + (1-\alpha) z_t, \end{pmatrix} \tag{3}$$

where *x, y, z* have replaced *D, S, Ď* respectively and with parameters' restrictions like in the original model. Moreover let us later denote $F = [f_1, f_2, f_3]$.

### 1.3. Some Basic Theory

**Corollary** (of the centre manifold theorem) [1, p.248]. Let us consider the one-parameter family of maps $G(k,u)$: $R \times R^2 \rightarrow R$ with $u=(x,y)$ and *G is in* $C^r$, $r \geq 5$. Let (0,0) be a fixed point and *DG*(0,0) be Jacobian of *G*. Then using centre manifold theorem, we find one-dimensional map $f_k$ defined on the center manifold and following statement can be deducted: if *DG*(0,0) has an eigenvalue equal to 1, then we have a local bifurcation. According to [1] the corollary can be extended to higher dimensions.

# 2. Main Results

## 2.1. Fixed Points and Their Properties

There are two fixed points of the map *F* defined by (3) and they are of the form:

$$\begin{aligned}(\bar{x}_1, \bar{y}_1, \bar{z}_1) &= (\bar{x}, T, \bar{x}) \\ (\bar{x}_2, \bar{y}_2, \bar{z}_2) &= (0, \bar{y}, 0)\end{aligned},$$

for any given $\bar{x}$ and $\bar{y}$.

Moreover it can be observed with numerical simulations that trajectories converge to the points of the form above defined and with following particular values:

$$\begin{aligned}\bar{x} &= \alpha(y_0 - T) \\ \bar{y} &= y_0\end{aligned}.$$

Convergence to the one or another fixed point depends on relation between initial values and parameters. In figure 1 convergence to the first fixed point is graphically presented.

Since at least one eigenvalue of the Jacobian of the map (3) at both fixed points is equal to 1, it can be stated that both fixed points are nonhyperbolic. Hence the Hartman-Grobman theorem does not apply. Based on corollary in section 1.3 there occurs a local bifurcation.

## 2.2. Certain Business Case

The model with a given set of parameters presented in the table 1 describes the particular business case of the sell-out of a product, depicted in the figure 1.

| | |
|---|---|
| $x_0$ | 100 |
| $y_0$ | 1000 |
| $z_0$ | 0 |
| A | 0,82 |
| T | 600 |
| k | 0,11 |
| α | 0,84 |

**Tab. 1.** Sample set of parameters' value to describe a real business case.

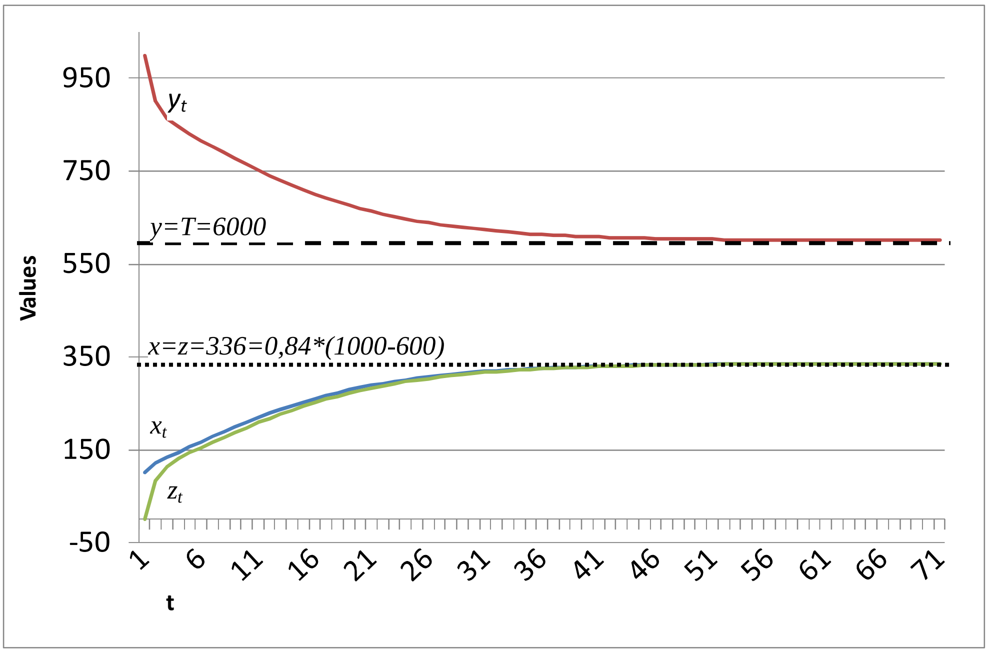


**Fig. 1.** Graphical interpretation of a model with parameters from table 1.

### 2.3. Bifurcation Diagrams

Parameters $A$, $\alpha$, $T$ and initial values of variables remain as they are presented in the table 1. Parameter $k$ representing price elasticity in general theory of economy and here it controls influence of overstock value on a given discount and through that also on a demand and inventory.
That influence of parameter $k$ can be observed on bifurcation diagrams presented in the figures 2, 3, 4, in regard to demand $x_t$, inventory $y_t$ and deliveries $z_t$ respectively.
Until $k \approx 1{,}22$ trajectories converge to the fixed point no. 1. Very interesting behavior can be observed from $k$ around 1,22. With numerical and graphical approach it can be suspected that there occurs chaotic behavior and for some ranges of parameters (e.g. around 1,38; 1,68; 1,90; 2,10) the trajectories oscillate in cycles of different periods (e.g. for $k$=1,38 with period 15; for $k$=1,68 with period 8).

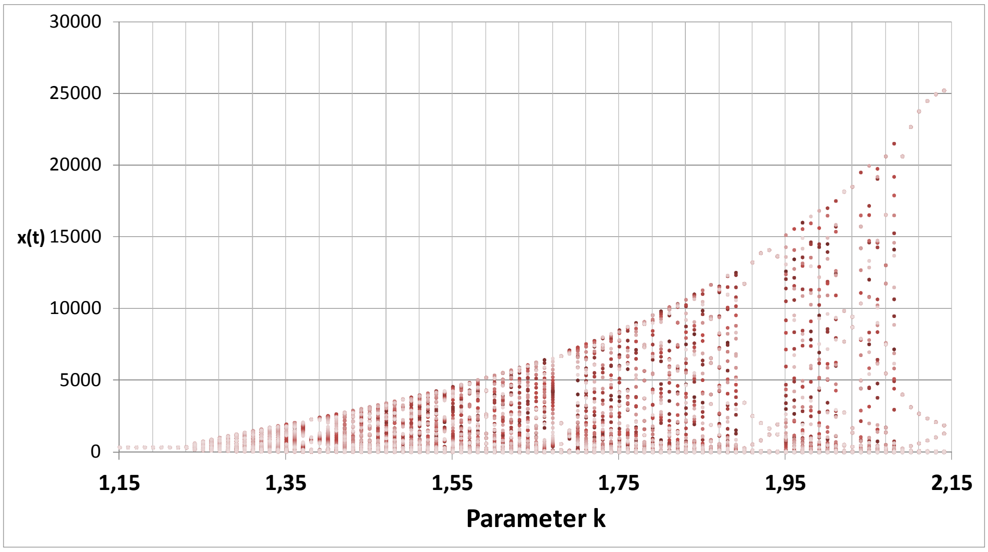


**Fig. 2.** Bifurcation diagram of variable $x_t$.

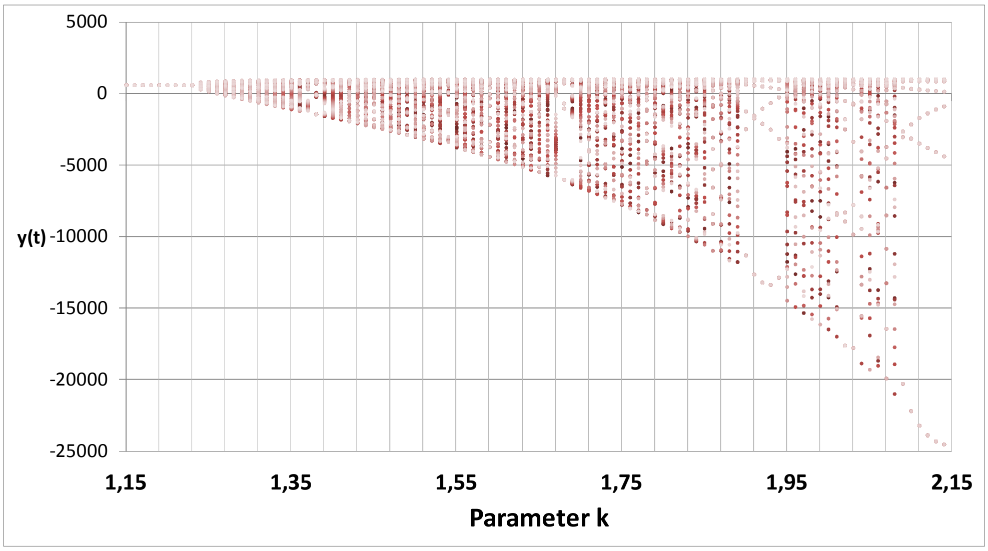


**Fig. 3.** Bifurcation diagram of variable $y_t$.

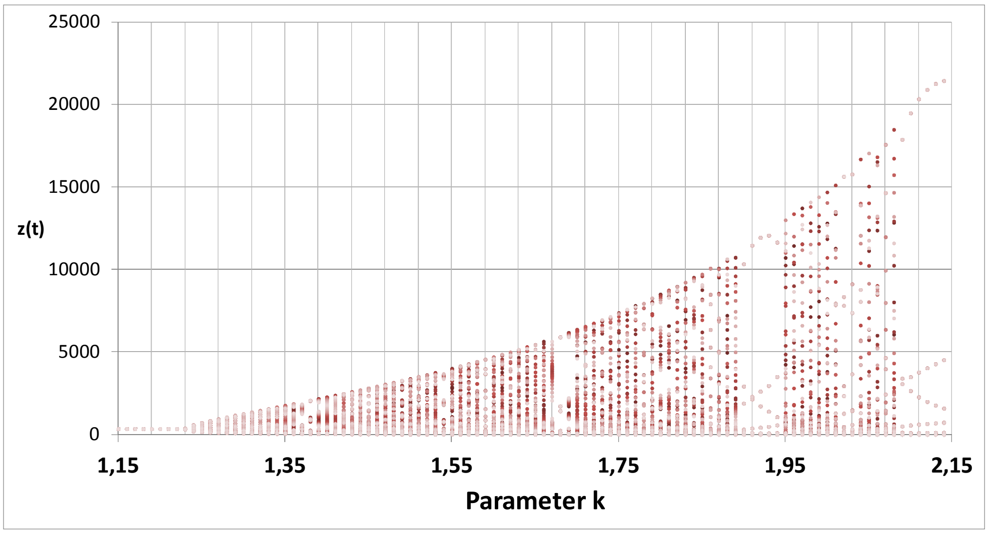


**Fig. 4.** Bifurcation diagram of variable $z_t$.

## 3. Technical Notes

The graph and bifurcation diagrams are prepared in MS Excel 2010. Bifurcation depicts trajectories values for 100 iterations starting from instance $t$=14 400.

## 4. Conclusions

The interesting asymptotic behavior of the model representing real business case have been presented in the paper. Although the model itself can be seen as relatively simple, thanks to the nonlinearity involved in the first equation it reveals course of the trajectories which is interesting and unexpected at the first glance.

The intention of the authors is to study the behavior of the model with analytical methods, particularly relying on the center manifold theorem, which is assumed to come as result of further studies of economical models.

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# Homotopy Perturbation Method and Trefftz Functions in Solving Two-Dimensional Wave Equations Describing Vibrations of the Membrane

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**Abstract.** The Trefftz functions method has been developing very quickly recently. The paper presents the application of this method to solving two-dimensional wave equation which describes vibrations of the membrane with variable thickness. The differential operator is resolved into two parts. The first one describes vibrations of the membrane with constant thickness. The second contains the rest of the original operator and is treated as inhomogeneity for the first one. Homotopy perturbation method (HPM) is used to calculate the successive approximation of the exact solution. The quality of the approximate solution is verified on the test example. The results obtained by HPM for the linear differential equation are also compared with those results obtained by method of successive approximation (also known as Picard's iterations). The comparison shows a complete agreement between results. The presented example shows the usefulness of the method.

**Keywords:** homotopy perturbation method, Trefftz function, wave equation, vibrations of the membrane.

## 1. Introduction

The Trefftz method was described for a first time by Erich Trefftz in 1926 in the paper [1]. Then this method has been developed by Herrera and Sabina, Jirousek, Kupradze, Zieliński and Zienkiewicz in [2, 3, 4, 5]. In this paper, the wave polynomials (Trefftz functions for wave equation) will be used. Papers [6, 7] contain a wide description of wave polynomials, their properties and formulas. In these papers also the method of wave polynomials and the convergence of the method are described.

The Trefftz method is used for solving various problems described by partial differential equations. The basic principle of this method is approximating the solution by a linear combination of the functions which satisfy the given equation identically. The coefficients of the linear combination are calculated based on known initial and boundary conditions. The commonly occurring vibrations are described by the wave equation in [8]. Vibrations of the membranes are described by a two-dimensional wave equation.

In the paper the method of solving linear problems of vibration of the membrane with variable thickness is described. In the second section the homotopy perturbation method (HPM) is also discussed. The example is presented in the third section. What is more, the third section contains a comparison of the results obtained by using HPM for the linear differential equation with the results obtained in the paper [9] by method of successive approximation (also known as Picard's iterations). After the conclusion in the fourth section the references are given.

## 2. Presentation of the Method

Let us consider the equation:

$$A(u) - f(r) = 0 \qquad \text{for } r \in \Omega, \tag{1}$$

where $A$ is a partial differential operator and $\Omega$ is a bounded subset of $R^n$, $f(r)$ is a known analytic function and $u$ is unknown function. If a direct problem is considered, (1) should be completed by the initial boundary condition:

$$B(u, \frac{\partial u}{\partial n}) = 0 \qquad \text{for } r \in \Gamma,$$

where $B$ is a boundary operator and $\Gamma$ is the boundary of the domain $\Omega$. In the case of nonlinear problems described by partial differential equation, the operator $A$ is nonlinear. Suppose that the operator $A$ can be decomposed into a simple (the solution can be found easily) linear part $L$ and the rest in accordance with the formula:

$$A = L + N.$$

Then (1) has the form:

$$L(u) + N(u) - f(r) = 0. \tag{2}$$

Considering the HPM [10], a homotopy $v(r,p)$ can be constructed:

$$H(v,p) = (1-p)[L(v) - L(v_0)] + p[A(v) - f(r)] = 0, \tag{3}$$

where $p \in [0,1]$ is embedding parameter and $v_0$ is initial approximation of (1). Applying the HPM, we can assume that the solution of (3) can be expressed as a power series in $p$, i.e. $v(r,p)$ can be expanded into a power series with respect to the parameter $p$:

$$v(r,p) = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \cdots. \tag{4}$$

Then:

$$u(r) = \lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots. \tag{5}$$

The approximate solution of (1), therefore, can be obtained:

$$u_N(r) = \sum_{n=0}^{n=N} v_n.$$

The convergence of this method has been proved by J. H. He in [11]. By inserting (4) into (3), we obtain equations for $v_0$, $v_1$, $v_2$, … Then, equations for $v_0$, $v_1$, $v_2$, … are solved by using Trefftz method (see example).

## 3. Example

In the paper [9] the following problem describing vibration of a membrane with variable thickness, was considered:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(f(x,y)\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(g(x,y)\frac{\partial u}{\partial y}\right) + Q(x,y,t), \qquad \text{for } (x,y,t) \in \Omega, \tag{6}$$

where $\Omega$ is a bounded subset of $R^n$.

$$Q(x,y,t) = 2x + \frac{9}{4}y + 4x^2t^2 - 4xt^2 + 2xyt^2 - 2xy^2t^2 - 2x^2 - \frac{9}{4}y^2 - 5xy - \frac{9}{2}yt^2 + 4x^2y + 5xy^2 + \frac{9}{2}y^2t^2 - 4x^2y^2, \tag{7}$$

$$f(x,y) = 1 - \frac{1}{4}x,\ g(x,y) = 1, \tag{8}$$

$$u(x,y,0) = u_0(x,y) = x(x-1)y(y-1), \tag{9}$$

$$\frac{\partial u(x,y,0)}{\partial t} = 0, \tag{10}$$

$$u(0,y,t) = u(1,y,t) = u(x,0,t) = u(x,1,t) = 0. \tag{11}$$

The exact solution of the problem described by (6) – (11) has the form:

$$u(x,y,t) = x(x-1)y(y-1)(1-2t^2).$$

The solution of the problem described by (6) – (11) has been obtained in [9] by method of Picard's iteration. In this paper the HPM is used to calculate the successive approximation of the exact solution.

According to (2) the equation (6) can be converted to the form:

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + \frac{1}{4}\frac{\partial u}{\partial x} + \frac{x}{4}\frac{\partial^2 u}{\partial x^2} = Q(x,y,t). \tag{12}$$

In the equation (12) the operators *Lu* and *Nu* have the forms:

$$Lu = \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2},$$

$$Nu = \frac{1}{4}\frac{\partial u}{\partial x} + \frac{x}{4}\frac{\partial^2 u}{\partial x^2}.$$

To solve (6), a homotopy should be constructed by applying HPM to (6) using (3). One obtains:

$$H(v,p) = (1-p)\left[\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 u_0}{\partial t^2} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2}\right] + p\left[\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} + \frac{1}{4}\frac{\partial v}{\partial x} + \frac{x}{4}\frac{\partial^2 v}{\partial x^2} - Q(x,y,t)\right]. \tag{13}$$

By choosing an initial approximate solution (9) and substituting (4) into (13) and rearranging the resultant equation based on powers of p-terms. One has:

$$p^0: \frac{\partial^2 v_0}{\partial t^2} - \frac{\partial^2 v_0}{\partial x^2} - \frac{\partial^2 v_0}{\partial y^2} - \frac{\partial^2 u_0}{\partial t^2} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} = 0,$$

$$p^1: \frac{\partial^2 v_1}{\partial t^2} - \frac{\partial^2 v_1}{\partial x^2} - \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial t^2} - \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial^2 u_0}{\partial y^2} + \frac{1}{4}\frac{\partial v_0}{\partial x} + \frac{x}{4}\frac{\partial^2 v_0}{\partial x^2} - Q(x,y,t) = 0, \tag{14}$$

$$p^2: \frac{\partial^2 v_2}{\partial t^2} - \frac{\partial^2 v_2}{\partial x^2} - \frac{\partial^2 v_2}{\partial y^2} + \frac{1}{4}\frac{\partial v_1}{\partial x} + \frac{x}{4}\frac{\partial^2 v_1}{\partial x^2} = 0, \tag{15}$$

.....

$$p^j: \frac{\partial^2 v_j}{\partial t^2} - \frac{\partial^2 v_j}{\partial x^2} - \frac{\partial^2 v_j}{\partial y^2} + \frac{1}{4}\frac{\partial v_{j-1}}{\partial x} + \frac{x}{4}\frac{\partial^2 v_{j-1}}{\partial x^2} = 0. \tag{16}$$

Solving the previous equations and considering the initial and boundary conditions (9) – (11) and results for $v_0$, $v_1$, $v_2$, …the approximate solution of (6) is in the form (5). By inserting (5) into (13), we obtain equations for $v_0$, $v_1$, $v_2$, … . Equations (14) – (16) are solved by using Trefftz functions as in [7]. The Trefftz functions $V_n$ are wave polynomials described in [7]. In the first step we have:

$$\frac{\partial^2 v_0}{\partial t^2} - \frac{\partial^2 v_0}{\partial x^2} - \frac{\partial^2 v_0}{\partial y^2} = 0.$$

Then:

$$v_0(x,y,t) \approx \sum_{n=1}^{N} c_n V_n(x,y,t) + w_p(x,y,t),$$

where $w_p(x,y,t)$ is the particular solution of nonhomogeneous equation (12). The particular solution is calculated according to the formula:

$$w_p(x,y,t) = L^{-1}(Q(x,y,t)).$$

In the considered case the source $Q(x,y,t)$ has a form of a polynomial. Therefore, the inverse operator for monomials has to be known. Let us denote $r_{kml}=L^{-1}(x^k\ y^m\ t^l)$, where $k,l,m=0,1,...$. Suitable formulas are presented in the paper [6]:

$$r^1_{kml} = -\frac{x^{k+2}y^m t^l - l(l-1)r^1_{(k+2)m(l-2)} + m(m-1)r^1_{(k+2)(m-2)l}}{(k+2)(k+1)},$$

$$r_{kml}^2 = -\frac{x^k y^{m+2} t^l - l(l-1) r_{k(m+2)(l-2)}^2 + k(k-1) r_{(k-2)(m+2)l}^2}{(m+2)(m+1)},$$

$$r_{kml}^3 = \frac{x^k y^m t^{l+2} + k(k-1) r_{(k-2)m(l+2)}^3 + m(m-1) r_{k(m-2)(l+2)}^3}{(l+2)(l+1)}.$$

For calculations we take:

$$L^{-1}(x^k y^m t^l) = \frac{1}{3}\left(r^1(x^k y^m t^l) + r^2(x^k y^m t^l) + r^3(x^k y^m t^l)\right).$$

Coefficients $c_n$ are chosen so that the mean square error of fulfilling initial and boundary conditions by an approximate solution is minimized. In the next step the equation has the form:

$$\frac{\partial^2 v_1}{\partial t^2} - \frac{\partial^2 v_1}{\partial x^2} - \frac{\partial^2 v_1}{\partial y^2} = -\frac{\partial^2 u_0}{\partial t^2} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} - \frac{1}{4}\frac{\partial v_0}{\partial x} - \frac{x}{4}\frac{\partial^2 v_0}{\partial x^2} + Q(x,y,t) = 0. \qquad (17)$$

Now an approximate solution is given in the form:

$$v_1(x,y,t) \approx= v_0 + \sum_{n=1}^{N} c_n V_n(x,y,t) + w_p(x,y,t).$$

$c_n$ and the particular solution are calculated as before. The next steps are similar to second step but in (17) there is no *Q(x,y,t)* and solution is a sum of previously calculated *v*, linear combination of Trefftz functions and a particular solution for a nonhomogeneous equation.

Because the exact solution is known, the quality of the approximation can be checked using two kinds of the error. The first one is calculated for point *x=y=0.5* (middle point of the membrane):

$$E_m = \sqrt{\frac{\int_0^{\Delta t} [u(0.5,0.5,t) - v(0.5,0.5,t)]^2 dt}{\int_0^{\Delta t} [u(0.5,0.5,t)]^2 dt}} \cdot 100\%.$$

The second error is calculated for the entire domain:

$$E = \sqrt{\frac{\int_0^1 \int_0^1 \int_0^1 [u(x,y,t) - v(x,y,t)]^2 dxdydt}{\int_0^1 \int_0^1 \int_0^1 [u(x,y,t)]^2 dxdydt}} \cdot 100\%.$$

The values of the errors $E_m$ and *E*, depending on the step number and the number of wave polynomials *n*, used in approximation are presented in Tab. 1 and Tab. 2, respectively.

| Number of polynomials | Number of steps | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 25 | 5.6904 | 1.0421 | 0.8733 |
| 36 | 5.6978 | 0.1097 | 0.0399 |
| 64 | 5.8591 | 0.1976 | 0.0091 |
| 81 | 5.8581 | 0.1725 | 0.0111 |

**Tab. 1.** A mean relative error of approximation ($E_m$[%]) for *x=y=0.5*.

| Number of polynomials | Number of steps | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 25 | 16.5528 | 13.3761 | 13.2998 |
| 36 | 6.5954 | 0.5306 | 0.1314 |
| 64 | 6.7586 | 0.5534 | 0.0649 |
| 81 | 6.7833 | 0.5695 | 0.0734 |

**Tab. 2.** A mean relative error of approximation (E[%]) in the entire domain.

In general the error of approximation decreases after each step. In these tables there are no results for $v_0$ because in this step time is not taken into account, and error is relatively big. We can observe a significant improvement in approximation when the number of wave polynomials is greater than 36. The value of the error after three steps (both for point $x=y=0.5$ and for entire domain) is smaller than 0.2%, which proves a very good quality of the approximation. When the number of wave polynomials is greater than 64 the both errors are almost constant. Therefore, a greater number than 64 wave polynomials used to approximation does not affect the improvement of the accuracy of approximation.

In Tab. 3 and Tab. 4 the results obtained in [9] by using Picard's iterations are presented.

| Number of polynomials | Number of iterations | | |
|---|---|---|---|
| | 1 | 2 | 5 |
| 25 | 0.7430 | 0.7214 | 0.7147 |
| 36 | 0.6468 | 0.0686 | 0.0112 |
| 64 | 1.1624 | 0.0768 | 0.0043 |
| 81 | 1.1374 | 0.0824 | 0.0021 |

**Tab. 3.** A mean relative error of approximation ($E_m$[%]) for $x=y=0.5$.

| Number of polynomials | Number of iterations | | |
|---|---|---|---|
| | 1 | 2 | 5 |
| 25 | 13.0272 | 13.3154 | 13.6519 |
| 36 | 0.8758 | 0.1464 | 0.0216 |
| 64 | 1.0380 | 0.1709 | 0.0080 |
| 81 | 1.2981 | 0.1859 | 0.0052 |

**Tab. 4.** A mean relative error of approximation (E[%]) in the entire domain.

As you can see, the results obtained by using Picard's iterations are better than results obtained by using HPM, but the differences are small. Fig. 1 shows the approximation by 64 wave polynomials (dot line) for $x=0.5$, $y=0.5$ after a) 1, b) 2, c) 3 steps and the exact solution (solid line) obtained by using HPM.

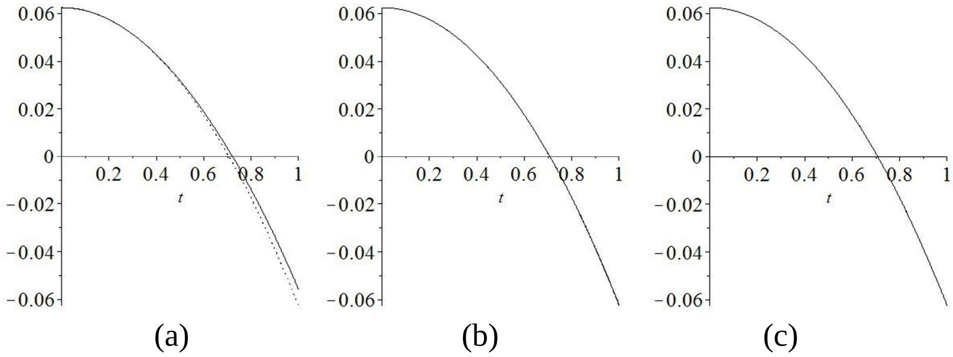
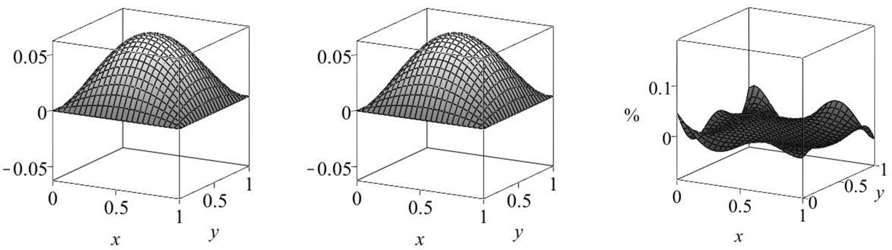



**Fig. 1.** The exact solution (solid line) for $x=0.5$, $y=0.5$ and the approximation by 64 wave polynomials (dot line) after a) 1, b) 2, c) 3 steps.

Fig. 2 shows the exact solution, approximation by 64 wave polynomials after 3 steps and the error of approximation for $t=0$ obtained by using HPM.




(a) (b) (c)

**Fig. 2.** The solution for $t=0$: a) exact, b) approximation after 3 steps, c) relative error of approximation [%].

Fig. 1 and Fig. 2 shows a very good accuracy of the approximation.

## 4. Conclusions

In recent times the Trefftz method is quite popular. The Trefftz functions are really useful in solving problems described by partial differential equations. In this paper these functions have been used to solve wave equations which describe the vibrations of the membrane with variable thickness. Successive approximations are obtained using homotopy perturbation method. An approximate solution consists of a linear combination of Trefftz functions and a particular solution for a nonhomogeneous equation. The inhomogeneity is related to the variable thickness of the membrane. The presented method gives very good results for a membrane of varying thickness. In this case the error decreases with the increasing number of polynomials and steps. An acceptable approximation is obtained already in the third step even for a small number of wave polynomials. In this paper only direct linear problems have been discussed. Although, the same method can be used for nonlinear direct and inverse problems.

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# Foreign Languages for the Labour Market Needs

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**Abstract.** Being able to use two foreign languages effectively, being competent and well–prepared for the today´s labour market needs has become EU top priority. The paper promotes some EU projects, looks closely at language preparation at universities and brings some ideas and tips on how to improve efficiency in language teaching when using authentic materials.

**Keywords:** languages, skills, competences, labour market, employers, authentic materials.

## 1. Language Policy and Learning Languages

Whereas every EU country is responsible for its own education and training system, EU policy under Education and Culture Committee of the European Commission supports national action in every country and addresses common challenges today´s Europe has to face such as ageing society, lack of skills in the workforce and global competition. Its objectives to address these challenges by 2020 are focused on lifelong learning and mobility, quality and effective education and learning, equity, social cohesions and active citizenship, creativity, innovation and entrepreneurship at all levels of education and training. Cooperation of working groups aims at mutual learning, education at all levels and for all age groups, exchange of good practices and fostering national reforms as well as developing EU–level tools [1].

One of the activities of the European Commission and its branch, the Directorate General for Education and Culture (EAC) in the field of languages is framed by Communication on Multilingualism (*“the more languages you know, the more of a person you are”*, Slovak proverb) and the Education and Training 2020 Strategy (ET2020). EAC´s responsibilities in the field of languages are: to support language learning across Europe and to promote linguistic diversity. Its policy is evidence–based and it also manages initiatives in support of language learning across Europe, notably under Erasmus+ programme, the new EU programme for Education, Training, Youth and Sports for 2014–2020 suitable for organisations and individuals (students, teachers, professionals, etc.) [2]. The programme itself wants to help tackle socio–economic changes today´s Europe has to face, e.g. risks of being unemployed after successful gradations and/or leaving school/apprentice lacking skills required by the labour market. A suitable candidate/graduate if he/she wants to achieve personal fulfilment on the EU job market should be an active player in society, should be well prepared for the labour market and economy needs, should be cultural aware and fluent in two foreign languages, able to communicate effectively, and equipped with enough skills and competences required to perform his/her job. Participation at programmes such as ERASMUS+ can help a lot [3]. The applicants train their specific discipline according to their field of study abroad and also improve their language and intercultural communication skills [4] and with such international experience they can and will undoubtedly perform better on the job market later in their professional career.

In coherence with the Europe 2020 strategy for growth, jobs, social equity and inclusion the aim of lifelong learning and education is to use human talent and social capital effectively. In the field of language preparation at schools (formal, non–formal, etc.) the objective is to prepare students to be able to use foreign languages effectively in the real world outside his/her classroom and so to make them more attractive to their future employers. Being skilled, fluent and competent, and with international experience, will help them find and safeguard their future positions at work. Strong language, multilingual communication and intercultural skills are very important not only for individuals but also for businesses.

## 2. Some Useful Tips and Ideas on the Use of Authentic Materials in Class

Once a university has to and wants to guarantee the right of every single learner to a good quality education one should also focus on the personality of the teacher/trainer providing the access to education him/herself. If the educational process should be good and quality, so must be the personality of the teacher/trainer/lecturer/presenter [5] and the teaching materials he/she uses.

If the teacher wants to keep up–to–date and his aim is to prepare his/her students for the requirements of the today´s labour market he/she should also upgrade his teaching materials. There is already a trend towards specific job–related training and away from general business English. Work with authentic materials is one of the possible (not always easy) ways. There are many possible sources of authentic materials: Teachers can obtain them quite easily directly from possible future employers, factories, businesses, company´s websites, etc., from subject teachers as well as from students themselves at the same school/university or from the Internet and/or local area. These materials are up–to–date, easily available, work–related, stimulating, but when used in class they must always be adapted for the education needs, fulfilling students´ needs and above all meeting specific students´ and labour market needs, which is hard work of the teacher him/herself. Some materials might be too general, too specific, highly technical and hard to understand, long and demotivating for students; their adaptation by the teacher him/herself might be time consuming, etc. Possible and useful way may be cooperation among language teachers, subject teachers, students and their future employers: Mutual projects seem to be of vital importance. Language teachers even if they are experts in language are usually not experts in various fields of technology, business etc. and might feel not very competent when teaching new specific stuff in a foreign language to their students. A significant help has been done in this area so far, e.g. change of a curriculum for future teachers, fostering language preparation for subject teachers in abroad via mobility courses, using CLIL (Content and Language Integrated Learning) and CALL (Computer–Assisted Language Learning) approaches, giving lectures at home and abroad; there are plenty of publishing houses, teacher training institutes, language schools, ESP (English for Specific Purposes) methodology, etc. Cooperation of language teachers and subject teachers/experts at project activities, consultancy for subject teachers not trained in facilitating language learning or vice versa, preparatory, mobility and interdisciplinary courses should become everyday reality [5]. What is more, cooperation among ESP teachers/trainers/lectures providing language education on a tertiary level in mixed ability classes of adult learners at universities of a technical kind and players on the labour market: employers, organisations, trade unions, governments, etc. should be the future road we take. Not to forget that these players from the world of work contribute to forecast what skills are needed in different sectors, including projections for language skills. (Read on more about ESCO project classifying European Skills, Competences, Qualifications and Occupations that is a part of Europe 2020 Strategy; further there is EU Skills Panorama and CELAN – Network for the Promotion of Language Strategies for Competitiveness and Employability available at [4].)

One should take into consideration that every learner (e.g. one of electrical engineering) is as well a language learner, during his/her study he/she should develop his/her language, communicative and cultural competences, as well as his/her professional skills and attitudes. Work with up–to–date authentic materials from their possible future employers during their formal educational process may help them be well–prepared for the requirements today´s labour market places on future employees these days. Authentic materials are relevant to students, work–related, stimulating and above all offer real language in specific context. On the other hand they have to be adapted by teacher so that they can be presentable, understandable as well as exploited to a maximum degree for students´ profit (and then re–usable).

Here are some ideas and tips:

- Brainstorm ideas on the chosen topic: use mind maps, visuals, company websites, reports, brochures, publications, promotional materials, catalogues, multi–media materials, journals, newspapers, video documentaries, radio broadcasts etc.
- With the whole class look closely at the title, heading, subheadings of a reading article.
- Analyse numbers, define abbreviations that occur in the text, discuss what they refer to.
- Work on vocabulary: look for specific words in the article: synonyms, antonyms, word collocations, word partnerships, acronyms.
- Analyse trends, graphs, tables, SWOT charts; equip students with the necessary language.
- Check understanding via multiple choice questions, true or false statements, close/open questions, finding correct alternatives to complete sentences/paragraphs, information gap activities, interactive dictation, creation of a text summary; foster pair and group work, etc.
- Utilize students´ pre–knowledge of the topic, ask them for clarification of specific, technical issues of their field of study; do not concentrate on accuracy.
- Consult local environment and/or the Internet; compare documents on the same topic.
- Bring a possible future employer in class.
- Get students to talk: adapt texts for discussion, use role–plays, simulations, prepare presentations of students´ own projects, help them communicate freely and effectively without being accurate, focus on students´ specific needs and fluency.
- Use authentic texts for practicing grammar in specific context.
- Prepare students to use English in the real world outside his/her classroom and make them more attractive to future employers.

Some useful tips and ideas presented above the author gained during her stay at International Projects Centre, 7 Colleton Crescent, Exeter, Great Britain, in 2009 when she took part in further teacher training in Business English [6].

## 3. Conclusion

According to Language Policy of the Council of Europe the fundamental human right of every learner to unrestricted and fair access to good quality education logically goes hand in hand with the need to have enough good quality trainers, teachers, lectures, language policy makers, multipliers and other stakeholders in society functioning as facilitators and supporters of the learning process. The room for cooperation of experts in various fields is inevitable and wildly open. There are various projects and programmes available under EU, such as ERASMUS+. The objective to prepare students to use foreign languages in the real world outside his/her class, for the world of work, with focus on their specific needs and to make them more attractive to their future employers can be met via using authentic materials in class. The paper gives some ideas and tips on the issue.

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# Existence of Bounded Solution of Fourth Order Difference Equations

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**Abstract.** The aim of this paper is to present sufficient conditions for the existence of bounded solutions of fourth order neutral difference equation of the form

$$\Delta\left(a(n)\Delta\left(b(n)\Delta\left(c(n)\Delta\big(y(n)+p(n)y(n-\tau)\big)\right)\right)\right)+f\big(n,y(n)\big)=0,\quad n\in N=\{0,1,\dots\}$$

where $a,\ b,\ c,\ p\colon N\to R_+=(0,\infty)$, a function $f\colon N\times R\to R,\ \tau$ is a positive number, and $y$ is an unknown real sequence.

**Keywords:** Nonlinear difference equation, boundedness, neutral type.

## 1. Introduction

In the paper we consider a nonlinear four-dimensional neutral difference equation of the form

$$\Delta\left(a(n)\Delta\left(b(n)\Delta\left(c(n)\Delta\big(y(n)+p(n)y(n-\tau)\big)\right)\right)\right)+f\big(n,y(n)\big)=0,\quad n\in N \qquad (1)$$

where $\tau$ is a positive integer, $\Delta$ is the forward difference operator defined by $\Delta y(n)= y(n+1)-y(n)$, sequences $a,b,c,p: N\to R_+$ satisfy the following conditions

$$0<p(n)\le r<1,\qquad for\ all\ \ n\in N \qquad (2)$$

$$c(n)\le a(n),\qquad \text{for all large } n, \qquad (3)$$

$$\sum_{j=0}^{\infty}\frac{1}{c(j)}\sum_{i=j}^{\infty}\frac{1}{b(i)}<\infty. \qquad (4)$$

Moreover,

function $f\colon N\times R\to R$ is continuous and monotonic with respect to the second variable (5)

and satisfies

$$yf(n,y)>0,\qquad for\ all\ y\neq 0\ \text{ and }\ n\in N. \qquad (6)$$

By a solution of (1) we mean a sequence $y=(y(n))$ which satisfies (1) for all sufficiently large $n$. We consider only such solutions which are nontrivial for all large $n$.

Let us begin with recalling a basic definition and an existence theorem which will be used in this paper.

**Definition 1.** (*uniformly Cauchy subset*, [1]) A set $S$ of sequences $l^\infty$ is uniformly Cauchy if for every $\varepsilon>0$ there exists an integer $M$ such that $|X(i)-X(j)|<\varepsilon$ whenever $i,j>M$ for any $X=(X(n))$ in $S$.

**Lemma 1.** (*Arzel'a-Ascoli Theorem*, [2]) A bounded uniformly Cauchy subset S of $l^\infty$ is relatively compact.

**Theorem 1**. (Krasnoselskii's Fixed Point Theorem, [3]) Let $B$ be a Banach space, $S$ be a bounded, closed, convex subset of $B$ and let $T, F$ be maps of $S$ into $B$ such that $Fx + Ty \in S$ for every pair $x, y \in S$. If $F$ is a contraction and $T$ is completely continuous, then the equation $Fx + Tx = x$ has a solution in $S$.

The aim of this paper is to establish some sufficient conditions for existence of bounded solution of (1).

## 2. Existence of Bounded Solutions

For convenience, we set

$$\rho(n) = \sum_{j=n}^{\infty} \frac{1}{c(j)} \sum_{i=j}^{\infty} \frac{1}{b(i)}.$$

By $l^\infty$ we denote the Banach space of a bounded real sequences $\big(y(n)\big)$, $n \geq \tau$ with the supremum norm

$$\|y(n)\| = \sup_{n\geq\tau} |y(n)| < \infty.$$

**Theorem 2.**
If conditions (2) - (6) are satisfied and there exists some nonzero constant $\omega$ such that

$$\sum_{n=0}^{\infty} \sum_{i=0}^{n} \frac{1}{c(i)} |f(n,\omega)| < \infty, \tag{7}$$

then there exists a bounded solution $y$ of (1).

*Proof.*
Assume that (7) holds with $\omega > 0$ (if $\omega < 0$ the proof is similar). Set $d = \omega$ if function $f$ is nondecreasing in second argument and $d = \frac{2\omega}{1-r}$ if function $f$ is nonincreasing in the second argument. Let $n_1 \geq \tau$ be so large that

$$\rho(n) \leq \frac{1}{2} \quad \text{for all } \ n \geq n_1 \tag{8}$$

and

$$\sum_{n=n_1}^{\infty} \sum_{i=n_1}^{n} \frac{1}{c(i)} f(n,\omega) < d(1-r). \tag{9}$$

Let

$$S = \left\{ y \in l^\infty : \ \frac{d(1-r)}{2} \leq y(n) \leq d, \qquad \text{for } \ n \geq n_1 \right\}$$

be a bounded, convex and closed subset of the Banach space $l^\infty$.
In order to use Krasnoselskii's Fixed Point Theorem we introduce the maps $F, T\colon S \to l^\infty$ as follows

$$(Fy)(n) \coloneqq -p(n)y(n-\tau), \tag{10}$$

$$(Ty)(n) \coloneqq d - \sum_{i=n}^{\infty} \frac{1}{c(i)} \sum_{j=i}^{\infty} \frac{1}{b(j)} \sum_{k=j}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big), \tag{11}$$

for $n \geq n_1$.

Now, we will show that maps defined above satisfy the assumptions of Theorem 1. Firstly, we prove that $(F+T)(S) \subset S$. Indeed, if $y, \bar{y} \in S$, it is clear that

$$(F\bar{y})(n) + (Ty)(n) \leq d.$$

Furthermore, for $n \geq n_1$, by (2), (3), (7) and (8), we have

$$(F\bar{y})(n) + (Ty)(n) \geq -p(n)\bar{y}(n-\tau) + d - \rho(n) \sum_{k=n_1}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big)$$

$$\geq -dr + d - \frac{1}{2} \sum_{k=n_1}^{\infty} f(k, y(k)) \sum_{s=n_1}^{k} \frac{1}{c(s)}.$$

From (5) and (9), we obtain

$$(F\bar{y})(n) + (Ty)(n) \geq \tfrac{d(1-r)}{2}.$$

Thus $F+T$ maps $S$ into itself.

The task is now to prove that $F$ is a contraction mapping on $S$. In fact, by (10) we have

$$|(F\bar{y})(n) - (Fy)(n)| = p(n)|y(n-\tau) - \bar{y}(n-\tau)| \leq r|y(n-\tau) - \bar{y}(n-\tau)|$$

for any $y, \bar{y} \in S$ and $n \geq n_1$. Hence, since $r < 1$, we get

$$\|Fy - F\bar{y}\| \leq r\,\|y - \bar{y}\|.$$

It is easy to see that $T$ is continuous. Let $(y^{(m)}) \in S$ be a sequence such that $y^{(m)} \to y \ \ as \ \ m \to \infty$. Because $S$ is closed, $y \in S$. Now, by (8) and (11) we get

$$\left|\big(Ty^{(m)}\big)(n) - (Ty)(n)\right| \leq \frac{1}{2} \sum_{i=n_1}^{\infty} \left| f\left(i, y^{(m)}(i)\right) - f\big(i, y(i)\big)\right| \sum_{j=n_1}^{i} \frac{1}{c(j)}, \quad n > n_1.$$

Since

$$\lim_{m\to\infty} \left| f\left(i, y^{(m)}(i)\right) - f(i, y(i))\right| = 0$$

and

$$\left| f\left(i, y^{(m)}(i)\right) - f(i, y(i))\right| \leq 2f(i, \omega), \quad \text{for} \quad i > n_1$$

we see from Lebesgue's dominated convergence theorem that

$$\lim_{m\to\infty} \left\|\big(Ty^{(m)}\big)(n) - (Ty)(n)\right\| = 0.$$

This means that $T$ is continuous.

Finally, we need to prove that $T(S)$ is uniformly Cauchy. To see this, we have to show that given any $\varepsilon > 0$, there exists an integer $n_2 > n_1$ such that for $m > n > n_2$

$$|(Ty)(n) - (Ty)(m)| < \varepsilon,$$

for any $y \in S$. Indeed, by (3), (5), (7), (8) and (11) we have

$$\begin{aligned}
&|(Ty)(n) - (Ty)(m)| \\
&\quad = \sum_{i=n}^{m-1} \frac{1}{c(i)} \sum_{j=i}^{\infty} \frac{1}{b(j)} \sum_{k=j}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big) \\
&\quad \le \sum_{i=n}^{\infty} \frac{1}{c(i)} \sum_{j=i}^{\infty} \frac{1}{b(j)} \sum_{k=j}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big) \le \rho(n) \sum_{k=n_1}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big) \\
&\quad \le \frac{1}{2} \sum_{k=n_1}^{\infty} \frac{1}{a(k)} \sum_{s=k}^{\infty} f\big(s, y(s)\big) \le \sum_{k=n_1}^{\infty} f(k, \omega) \sum_{i=n_1}^{k} \frac{1}{c(i)}.
\end{aligned}$$

Using (7), it is clear that $T(S)$ is uniformly Cauchy. Therefore, by Theorem 1 and Lemma 1, there exists $y \in S$ such that $y(n) = (Fy)(n) + (Ty)(n)$ for $n \ge n_1$. This completes the proof.

□

**Example 1**
Let us consider the difference equation (for $n > 1$) of the form

$$\Delta\left(n^2 \Delta\left(n^2 \Delta\left(n^2 \Delta\left(y(n) + \frac{1}{2} y(n-1)\right)\right)\right)\right) + \frac{(12n^7 - 10n^6 + 12n^5 - 4n^2 + 1)(2n+1)^3}{2n^4 (n^2 - 1)^4 y(n)} = 0,$$

All assumptions of Theorem 2 are satisfied with $\omega = 2$. Hence the above equation has bounded solution. One of such solutions is

$$y(n) = 1 + \frac{1}{n^4}.$$

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# Potential Prosodic Errors in the English Pronunciation of Slovak Speakers

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**Abstract.** This overview concerns the suprasegmental features which are likely to cause problems in the English pronunciation of Slovak learners of English. The following prosodic features are introduced: stress, intonation, strong and weak forms, speaking rate, and rhythm. Moreover, the English suprasegmentals are contrasted with the Slovak ones, which may be useful to Slovak learners with regard to their English pronunciation.

**Keywords:** English pronunciation, pronunciation error, prosody, suprasegmental features.

## 1. Introduction

L2 learners of English are required to master both segmental and suprasegmental features if they wish to achieve clear, appropriate and comprehensible pronunciation. However, ample research supports the idea that prosodic features appear to be even more important than segmentals when it comes to intelligibility. Thus, it is vital that teachers and learners pay careful attention to stress, intonation, rhythm etc.

## 2. Suprasegmental Features

According to P. Roach's *Glossary of Phonetics and Phonology* [1], agreement about how many prosodic features there are has never been fully reached. But stress, intonation, pitch, loudness, tempo and rhythm are the most frequently mentioned features within suprasegmentals.

These features are of considerable importance within intelligibility between L2 learners and native speakers of a target language. J. Anderson-Hsieh et al. [2] conducted a study which compared a speaker having a good prosody + poor segmentals to a speaker having poor prosody, but possessing good segmentals. It was revealed that English native speakers evaluate the first type of speaker more positively.

Moreover, it seems that paying particular attention to suprasegmental features not only improves learners' intelligibility, but native speakers of English also seem more intelligible to L2 learners. The following suprasegmental errors are only a selection of a considerable number of possible errors Slovak learners are likely to make in their English pronunciation in terms of suprasegmental features.

- stress;
- intonation;
- strong and weak forms;
- speaking rate;
- rhythm.

## 2.1. Stress

Stress causes severe difficulties for Slovak students since it is markedly different from English stress. Slovak learners of English are often unaware of prominent syllables and the way these syllables determine stress. While stressing a syllable, the loudness and the length of vowels are increased or the pitch of the vowels is changed in some way [3].

"As is well known, English is not one of those languages where word stress can be decided simply in relation to the syllables of the word, as can be done in French, where the last syllable is usually stressed, Polish, where the syllable before the last – the penultimate syllable is usually stressed, or Czech, where the first syllable is usually stressed" [4]. Thus, it is likely that English native speakers detect a number of occurrences involving the inappropriate stress placement.

There exist rules which help determine the place where stress should be put. However, these may seem fairly complex. Therefore, we agree with the opinion that since English word stress is extremely difficult and complex matter for L2 learners in terms of prediction, it is best for them to think of the placement of stress as a property of an individual word to be learned when the actual word itself is learned [4].

We think that the issue of stress should be carefully approached by both learners and teachers since the incorrect stress placement can cause major problems regarding intelligibility, which may negatively affect communication.

## 2.2. Intonation

The two intonation systems of the both languages vary substantially. Hence, it may be difficult for L2 learners to understand it well and use the intonation properly in order to communicate effectively.

R. Pavlík (2000) highlights five major differences between the Slovak and English intonation:

- rise-fall and fall-rise are not normally present in one syllable in Slovak;
- particular tones have various functions in the two languages;
- in Slovak, sentences usually begin with a stressed syllable unlike sentences in English;
- tag questions intonation may be realized differently in Slovak and English;
- the range of pitch is considerably broader in comparison to the Slovak pitch.

In connection with this, Z. Kráľová [5] measured the minimum and maximum values of the basic tone F0 in the particular sentences. It was discovered that Slovak speaker's intonation values were significantly lower than the values of English native speakers.

Slovak learners need to bear in mind that altering the pitch while speaking allows speakers to convey subtle range of meanings. Therefore, intonation is regarded as a key factor in terms of speaking.

## 2.3. Strong and Weak Forms

In the English language, there is the occurrence of approximately 40 English words, which are frequently pronounced in their weak forms. These words belong to auxiliary verbs, prepositions, conjunctions etc. It is vitally important to mention that there exist contexts where only the strong forms are acceptable as well as other contexts where the weak form is considered to be normal pronunciation. Strong forms are normally used:

- at the end of the sentence;
- when a weak form word is being contrasted;
- for the purpose of emphasis;
- when a weak form word is being "quoted" [6].

It seems that Slovak learners of English do not always obey the rules regarding strong and weak forms. They tend to use only the full-form pronunciation due to the L1 interference.

We consider strong and weak forms to be of high importance, because these forms are not only included in speaking, but are also involved within listening. The majority of students have enormous problems with regard to understanding spoken text. They encounter intelligibility problems from the perception point of view. Thus, they need to be fully aware of such forms of words both in terms of production and perception.

### 2.4. Speaking Rate

English speaking individuals speak at an average rate between 152 words per minute to nearly 170 words per minute and the average length of segment is from 9 words per turn to 11 words per segment. It has been displayed that serious and important portions which can be not easily predicted are spoken at a slower rate. Older people have a tendency to speak at a slower rate and they obviously produce more variations within their turn's length. It has also been discovered that males speak at a slightly faster rate than females. However, the difference between the speaking rate of males and females is quite small [7].

According to H. Riggenbach [8], it is expressed that non-native speakers possess a slower speaking rate in comparison to native speakers.

### 2.5. Rhythm

English is characterized by an isochronic rhythm. There is a tendency of keeping the same amount of time from one stressed syllable to another one. As a result of this, English is in the category of stress-timed languages. In Slovak, syllables which are stressed do not occur at regular intervals of time – this is the reason why the syllable-timed rhythm of the Slovak sentence is different from the English sentence. The Slovak rhythm is determined by the amount and type of syllables within words [3].

We assume that Slovak speakers have a tendency to obey the rules of the Slovak rhythm and neglect the rules of the English rhythm.

What we consider to be highly important is the fact that, as we can see above, not only do most of these errors affect the production of speech, they also markedly influence the process of understanding. Intonation, stress, assimilation, elision or strong and weak forms are the features typical of English pronunciation. Many times, L2 learners know all the words in an utterance they hear, and yet are sometimes unable understand the words. The reason for this is that both the perception and production of suprasegmental features differ significantly in the two languages.

## 3. Conclusion

Being aware of prosodic features is of cardinal importance for Slovak learners of English, since they convey finer shades of meaning. Moreover, these features also play a major role in terms of perception. Word stress or weak forms can cause a lot of problems within listening comprehension of L2 learners. Thus, teachers of English should not neglect teaching suprasegmentals as they substantially influence both production and perception of speech.

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# Economical Application of Moment Equations

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**Abstract.** In this paper we are showing a possible application of the moment equations of first order. The bank account with randomly changing interest rate is discussed. The change of the interest rate depends on whether the client is treated to a special interest rate or not. As it is shown in the paper the coefficients of the equation are changing in accordance with a homogeneous Markov chain. Therefore we use moment equations to model the mean value of capital in every month. The growth of capital on the account is expressed by a difference equation that is transformed to a system of moment equations of first order. The system is solved and the effect on customers is discussed.

**Keywords:** Bank account, randomly changing interest rate, moment equation, mean value.

## 1. Introduction

The bank introduces a new product: an interest-bearing deposit account with an interest rate of $i\%$ p.m. and a duration of at least one year. In order to attract clients, the bank starts following marketing campaign: from all of the clients who open the account during the given period some of them would be awarded different interest rate $j\%$ p.m. higher than the guaranteed one. But those clients who are awarded higher interest rate in one month cannot be awarded it in the next one.

## 2. The Model of the Problem

Let $K_0$ denotes the capital at the beginning of the first month. We assume that the interest is credited to the account at the end of each month. The capital at the beginning of the second month is given as

$$K_1 = K_0 + U$$

where

$$U = K_0 \cdot \frac{i}{100}$$

and $i$ is the guaranteed interest rate in per cent. Similarly, we obtain the capital in the next month from the capital in the previous one as

$$K_{n+1} = K_n + U = K_n + K_n \cdot \frac{i}{100} = K_n \left(1 + \frac{i}{100}\right). \qquad (1)$$

Equation (1) represents homogeneous difference equation of first order. (More on modeling economic processes can be found in [1]).

In our model the interest rate is changing according to whether the client is chosen in the month or not. Let state $\theta_1$ represents the situation that the client is not chosen and state $\theta_2$ that he/she is. The change between states is made by jump at the end of the month (for simplicity we are assuming the length of the month is unified and equal to 30 days). Receiving or not receiving the special interest rate creates a series of random variables

$$\{\xi_n,\ n = 0, 1, 2, \ldots\}. \tag{2}$$

It is obvious that (2) is the series with discrete time and discrete states. Moreover, the transition from one state to another depends only on the state the system is actually in and not on the states it was in before. Therefore the series (2) will be considered as a discrete time Markov chain. (The properties of Markov chains can be found for example in [2])

Now we determined the Markov chain. Let $m$ be the number of clients that opened the account during the set period and $l$ be the number of clients who are randomly chosen for the higher interest rate,

$$m \geq 2l + 1.$$

The transitions between states $\theta_1$ and $\theta_2$ in every moment $n$ are set by transition probabilities $p_{i,j}(n, n+1),\ \ i, j = 1, 2;$ forming a transition probability matrix

$$\Pi(n+1) = \begin{pmatrix} \dfrac{m-2l}{m-l} & \dfrac{l}{m-l} \\ 1 & 0 \end{pmatrix}. \tag{3}$$

The rows of (3) denote the actual state and the columns next state, for example

$$p_{1,2}(n, n+1) = P\{\xi_{n+1} = \theta_2 \mid \xi_n = \theta_1\}. \tag{4}$$

Equation (4) denotes probability that you do not get special interest rate in this month but you will get it the next one.

As the rule is that the client cannot be given the higher interest rate in two following months, the probability

$$p_{2,2}(n, n+1) = 0$$

and obviously the probability of transition to state $\theta_1$ is sure event, so

$$p_{2,1}(n, n+1) = 1.$$

Also, every month can be chosen $l$ clients from only $(m-l)$ clients. Therefore, the probability of moving from state $\theta_1$ to state $\theta_2$ is

$$p_{1,2}(n, n+1) = \frac{l}{m-l}$$

and the probability of staying in state $\theta_1$ for the rest of clients is

$$p_{1,1}(n, n+1) = \frac{m-2l}{m-l}.$$

We assume that $m$ and $l$ remain constant in time. Thus, the transition probabilities are the same in every moment of jump which means they do not depend on parameter $n$. This allows us to write $\Pi$ instead of $\Pi(n+1)$.

The initial probability distribution for Markov chain (2) is

$$\overline{p}(0) = (p_1(0), p_2(0)) = \left(\frac{m-l}{m}, \frac{l}{m}\right)$$

where

$$p_k(0) = P\{\xi_0 = \theta_k\}, \quad k = 1, 2.$$

In our model equation (1) is of the form

$$K_{n+1} = K_n\, a(\xi_n) \tag{5}$$

which coefficients are changing as follows

$$a(\theta_1) = a_1 = \left(1 + \frac{i}{100}\right),$$

$$a(\theta_2) = a_2 = \left(1 + \frac{j}{100}\right)$$

in accordance with Markov chain (2).

Equation (5) belongs to the class of stochastic equations, called random ordinary difference equations with random structure or random parameters. Such equations are usually investigated using their trajectories, i.e. by solutions in every possible case that occurs. In the Fig. 1 there are shown some of the solutions of (5), i.e. the possible growths of capital $K_0$.

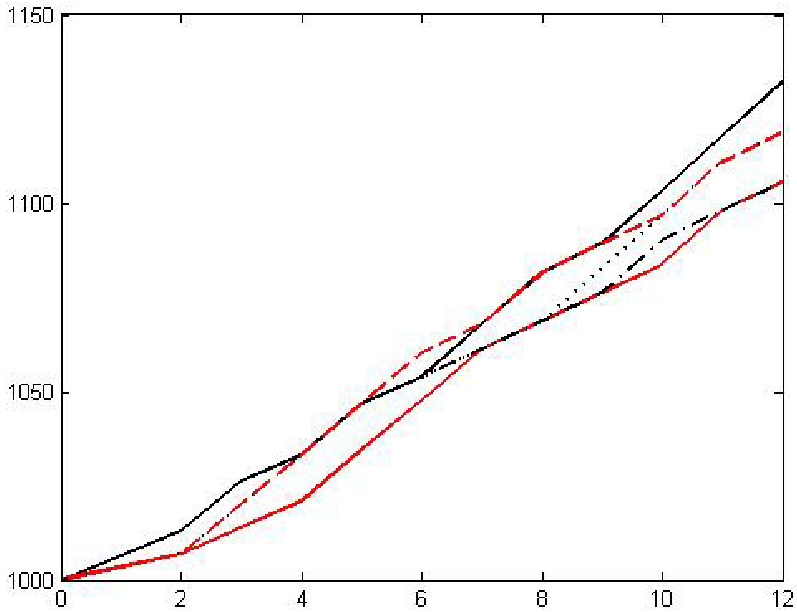


**Fig. 1.** Some **s**olutions of (10) for initial deposit 1000 €, duration 12 months and interest rates 0,7 % and 1,3%.

## 3. Solution of the Problem

From Fig. 1 it is obvious that the value of capital $K_n$ is different for different series (2). Therefore, it is impossible to calculate all of the growths of $K_n$. And it is not necessary. Instead of finding the solution of (5) we find the mean value $E(n)$ of capital $K_n$.

**Theorem** [3]: Let $(\Omega, F, P)$ be a probability space where $\Omega$ is a sample space, $F$ is set of all events (the σ-algebra) and $P$ is some probability measure. Let

$$x_{n+1} = a(\xi_n)\, x_{n,} \quad a(\xi_n) \neq 0, \quad n = 0, 1, \dots, \tag{6}$$

where $\xi_n$ is random discrete time Markov chain with two states $\theta_1, \theta_2$ and with constant transition probability matrix

$$\Pi = \begin{pmatrix} 1-\lambda & \lambda \\ \nu & 1-\nu \end{pmatrix}.$$

Then moment equations of first order for (6) are of the form

$$E_1(n+1) = (1-\lambda)a_1E_1(n) + \nu a_2E_2(n),$$

$$E_2(n+1) = \lambda a_1E_1(n) + (1-\nu)a_2E_2(n).$$

Here $a_1 = a(\theta_1)$, $a_2 = a(\theta_2)$. ■

According to the theorem the moment equations for (10) are

$$E_1(n+1) = \frac{m-2l}{m-l}a_1E_1(n) + a_2E_2(n), \quad (7)$$

$$E_2(n+1) = \frac{l}{m-l}a_1E_1(n). \quad (8)$$

Equations (7) and (8) are a system of homogeneous difference equations that are solved by standard methods (see [4]). The general solution is

$$E_1(n) = c_1a_2(A+B)^n + c_2a_2(A-B)^n,$$

$$E_2(n) = c_1(-A+B)(A+B)^n + c_2(-A-B)(A-B)^n,$$

$$c_1, c_2 \in R; n = 0,1,2,\dots$$

To calculate constants $c_1$, $c_2$ we use initial conditions

$$E_1(0) = a_1K_0p_1(0) = \frac{m-l}{m}a_1K_0,$$

$$E_2(0) = a_2K_0p_2(0) = \frac{l}{m}a_2K_0.$$

Now the mean value $E(n)$ of capital $K_n$ is

$$E(n) = E_1(n) + E_2(n).$$

In the Fig. 2 we see the mean value $E(n)$ for various numbers of chosen clients. With growing number of chosen clients, the mean value of capital is decreasing.

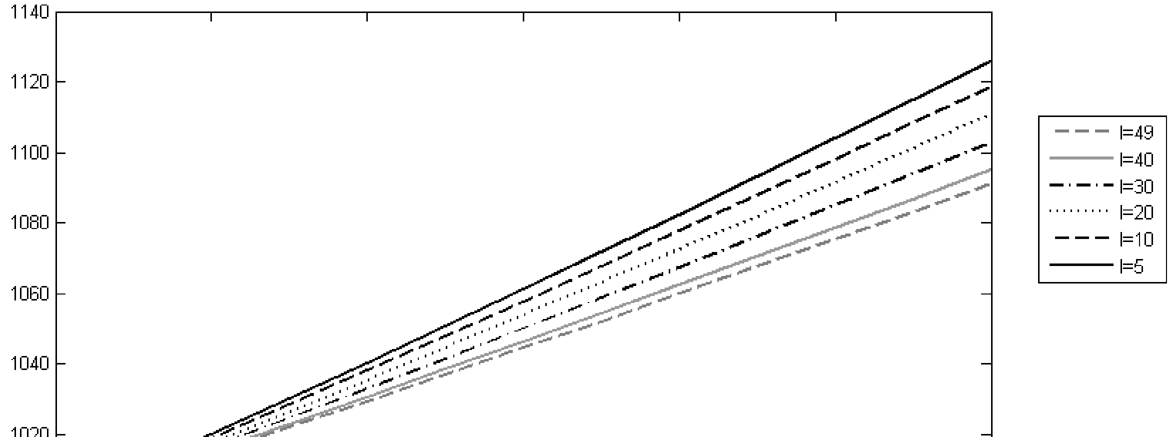


**Fig. 2.** The mean value of growth of the deposit $K_0$=1000 € for interest rates 0,7 % and 1,3 %.

## 4. Conclusion

In this paper we derived the equation modeling the growth of capital with random interest rate. To solve this problem we introduced the theory of moment equations. These equations represent a very useful tool in investigating difference equations with random character. In this case, they allowed us to calculate the mean growth of capital while neglecting the stochastic structure of the original equation. Using the solutions of moment equations we showed that the increasing number of chosen clients caused the decreasing mean value of capital growth. Similarly, we could model the situation under different conditions, for example when the number of chosen clients is constant but the number of involved clients is changing, and also for different combination of interest rates.

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# Image of Slovakia Abroad

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**Abstract.** The paper deals with the problem of Slovak image abroad. It mentions several studies dealing with this issue and points out at the factors that are involved in creating the nation brand as stated by S. Anholt. Moreover, it also discusses the ways our image should be promoted in the future. Slovakia is a small country in terms of population and geographical size and does not draw enough attention. That is why innovation of our image and brand strategy can be found crucial.

**Keywords:** image of Slovakia, nation branding, innovation, Simon Anholt, foreign press.

## 1. Introduction

In my doctoral thesis I plan to analyze all the texts related to Slovakia published by 5 British dailies, namely *The Guardian*, *The Daily Star*, *Daily Mirror*, *The Daily Telegraph* and *The Financial Times* in 2009-2014. In this way, besides other partial objectives, I hope to study factors creating our image in Britain and ways of influencing it.

Mass media are powerful image makers and their role in creating an image of a country cannot be underestimated. Image of a country is not an easily defined term and indeed is a very complex issue because there are too many unpredictable influences and variables to be considered.

Here we should also be careful not to mix up country image, product image and country of origin image. Country image can be understood according to I. Martin and S. Eroglu [12] as “a set of descriptive, inferential and informational beliefs about a country.”

As S. Anholt [2] claims, we cannot fully apply marketing terms and standards when it comes to the image or brand of a country. Countries cannot be compared to business companies because their behavior is different and do not really create any product in its strict sense. Life of any country is a subject to many events and changes and the state itself is a huge institution. Unfortunately, (at least for those who want to modify it), it is a relatively fixed idea and it is naturally really difficult to change it as people are not prone into leaving their fixed stereotypes.

## 2. How Can We Create an Image of a Country?

One of the prominent scholars that deal with the problem of country branding - S. Anholt [1], recognizes 6 aspects – indicators contributing to the country image formation. These are channels we use for communication with the outside world and employing them they can be turned into productive branding tools.

1. Tourism is one of the most powerful tools especially due to the power of personal experience.

2. Export brands. If the consumer is satisfied with the products, he tends to perceive the country of origin/production favorably as well and vice versa.

3. Government policy, especially the ability to cope with problems e.g. the unpredictable ones such as the natural catastrophes.

4. Community of traders. Here, it is considered how attractive the respective country is for potential businessmen and investors and how the foreign companies prosper there.

5. Cultural exchange and export of culture. This includes the way the country is depicted in various movies and books. As M. Držka [7] claims , good image of New Zealand was enhanced by the success of the Lord of the Rings trilogy whereas the image of Slovakia was certainly not helped by the way our country way is shown by the films as *Eurotrip* or *Hostel*.

6. Citizens of a country. Their behavior abroad can often substantially contribute to the country image. Besides ordinary citizens, the image of a country can be supported by well-known sportsmen, artists etc.

By careful consideration of all the “channels” mentioned above, efficient planning and targeted campaigns we can certainly improve the image of any country. However, the perception of other country often reflects the way we perceive ourselves (our country). Many theories based in social psychology claim that we even cannot perceive ourselves properly without a certain level of knowledge about and interaction with other countries [11]. Several contemporary researchers claim that we tend to judge other countries from the point of view of our own interests [11]. They also suggest that the stereotypical perception of other nation or a country is inversely proportional to the information and knowledge about the given country or a nation.

The study of I.Kleppe and L.Mossberg [11] also confirms that the country image shows unique qualities in every target country, depending on the perceived importance of the other country. That is why we should be really careful with applying so called generic images of countries. As I. Kleppe and L. Mossberg and Brysk et al. [11] further claim, the perception of a country in another country might be influenced by possible historical ties between countries, (similar to family relationships, e.g. younger and older brother), especially when there is a common historical background.

## 3. Slovak Strategies of Creating the Country Image – Space for Innovation?

According to information available, the only one study focused on our image in solely British print media was conducted by British scholar A. Burgess in 1997 and its title is *Writing Off Slovakia to "the East"? Examining Charges of Bias in British Press Reporting of Slovakia, 1993-1994.* The author studied the way British press reported about Slovakia in 1993-1994, i.e. just after the split of Czechoslovakia. The 3 most frequently reported topics were the “ Velvet divorce“ and its consequences, the Gabcikovo/Nagymaros dam dispute and the Hungarian minority rights. A. Burgess (1997, p. 679) noticed that there was noticeable bias in coverage of Slovakia in British media but he concluded that this was not due certain prejudice against Slovakia but due to the fact “after the collapse of communism , a discourse developed which understood there to be *East* defined by proclivity for intolerance, extremism and ethnic conflict.“ In other words, our country was in the shelf labelled Eastern Europe with all the implications. However, as the author further notes, concerning the Gabcikovo/Nagymaros dam controversy and the controversy over Hungarian minority rights, Slovakia came off worst[6].

As the title of K. Henderson´s study *Slovakia- The Escape from Invisibility* indicates, Slovakia was virtually invisible to foreign media for a long time. Our country first hit the headlines in 1991 because of the problem of Slovak nationalism. The news focused on small-scale nationalist demonstrations in the central square in Bratislava and unfortunately, the image of Slovaks depicted as "faintly ridiculous extremists" stayed for a long time afterwards. K. Henderson also suggests that the personality of V. Mečiar contributed heavily to the bad image of Slovakia abroad. However, the image of Slovakia improved considerably in the mid - and late 1990s. K. Henderson further claims that after 2000 Slovakia is no longer invisible [10].

A. Školkay [15] in his analysis of foreign correspondents´ activities in Slovakia determines the most frequent topics related to Slovakia in the foreign media, namely the Roma minority and their related problems, Hungarian and partially also a Jewish minority, Slovak economic affairs and migration. He estimates the number of foreign correspondents covering Slovakia to be 20. Most of them are temporarily or permanently based in Slovakia, some of them are ethnic Slovaks and a large part of them reside in neighboring countries.

The occurrence frequency of the topics mentioned above can be confirmed by the analysis of A. Salner and M. Beblavý [14] that studied the image of Slovakia in several prestigious international mass media in 1989-1999.

The central problem of our image can be worded in a very simple way: They do not know much about us at all. However, if taken positively, this also means that the blank space can be filled with positive content.

Unfortunately, the INEKO survey [17] that studied our image in 2005-2007 in chosen print media, namely *The Economist*, *Financial Times*, *The New York Times* and *Le Monde* showed that there is a noticeable trend in increasing negative reporting of Slovakia.

According to the analytical report of INEKO researchers O. Gyarfášová, M. Bútora and Z. Bútorová [9] that studied the image of Slovakia in chosen foreign media, primary associations the interviewed media experts had with Slovakia were: strong story of the country (our successful integration into Euro-Atlantic structures, fight against Mečiarism), proximity (propinquity) and potential. The unsatisfactory brand profile can be considered the main weakness of our image.

In 2011 Slovakia joined The Anholt-Gfk Roper Nation Index that shows how the selected countries are perceived abroad. In this rather extensive research (in 50 countries with 20,000 respondents) Slovakia ranked 38th, which only confirmed the fact that Slovakia is perceived as an unknown country with no clear associations to be connected with [9].

The Slovak Tourist Board (SACR) is also trying to improve our country´s image. In 2011 this agency conducted an analysis of Slovak marketing strategy in the field of tourism (in 18 countries worldwide). This analysis clearly states that our most noticeable weaknesses are the entrepreneurial environment and the tourism infrastructure. Our brand index value is too low (3.58) which indicates we are not perceived as an attractive holiday destination. It is an imperative to develop basic brand awareness in the target and potentially attractive markets. Considering our availability of financial resources the activities have to be really target-oriented communication activities [16].

S. Anholt [1] also studied the level of attractiveness and interest of country presentation to external audiences and he found out that, simply said, old boring things are extremely boring, new boring things are still boring, old interesting things are fairly entertaining but new interesting things are extremely entertaining. That is why if the presentation of our country is to be interesting and captivating it is supposed to get rid of old boring ways of presentation and focus on the innovative ones. Here, innovation is the key to successful presentation. We should definitely focus on redefining old stereotypes in our country presentation, typically including Juraj Jánošík, folklore, sheep cheese, etc., because they do not really create any special interest. Even these stereotypes perhaps deserve unusual, original presentation.

M. Držka [7] supposes that according to this Anholt´s classification the presentation of our culture and traditions will fall in the range of extremely boring to fairly entertaining and we should definitely come with new concepts and ideas. Among others, he suggests promoting our achievements in science and technology, e.g. the ESET company according to the Finnish model of promoting NOKIA in the 1990s. Our competitive identity can also be enhanced by promoting the achievements of our sportsmen, e.g. ice-hockey players.

A very well- prepared analysis of several Slovak experts *Slovakia-Country with Potential* [4] attempts to redefine our national image, and boldly enough, tries to capture the very essense of being Slovak, features of our national identity and mentality. If our presentation abroad is to be successful, it has to be authentic and credible. The analysis lists our cultural myths and stereotypes in an interesting way and in the presentation of our country it proposes focusing on presenting Slovakia as:

- a country that is developing rapidly and due to this it offers many opportunities;
- a unique, original and a culturally genuine country;
- a concentrated central Europe;
- a country of innovative and progressive people that think out of the box;
- a country able to adapt to harsh conditions;
- a country of accumulated energy: country of creative and interesting people;
- a country of positive surprises.

At present the need of innovation of our country presentation abroad is trying to be met by the initiative of our Ministry of Foreign and European Affairs that created the website www.brandingslovenska.com with the aim of collecting ideas for a more efficient presentation of Slovakia abroad. As stated by R. Boháč, Director General at the Slovak Ministry of Foreign and European Affairs, the brand can only become successful if it is well accepted by the domestic audience and the people in Slovakia can identify with the presentation concept [5]. In 2015 a completely new branding strategy with new visuals (logo, etc.) will be created. It may be interesting to observe whether this new presentation will be reflected in the Slovak image improvement abroad.

## 4. Conclusion

The image of Slovakia abroad, especially in foreign media, is not particularly favorable. Our country does not have a strong brand and its image does not feature any strong associations. The press coverage mostly focuses on issues related to minorities, economic problems and economic migration. Our image should therefore be redefined to become more authentic and attractive. It is, however, a complex issue that deserves deeper analysis and cannot be solved by instant solutions. Country branding is a rather a long-lasting process that requires hard and dedicated work of public diplomacy, PR, and support of various state institutions.

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# Using Video in English Language Teaching

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**Abstract.** This paper promotes a creative and diverse use of video as a tool that helps students learning English language. We describe several video types and video teaching techniques in it. Video making activities are also included as they can serve as an inspiration for other teachers when preparing to the particular lesson. Teachers can choose from different techniques and adjust them so as they fit to their students and available classroom equipment.

**Keywords:** video, language, teaching, type, technique, classroom.

## 1. Introduction

The purpose of this paper is to provide information about effective use of video in foreign language teaching. The inclusion of video materials in teaching is another way to get students engaged and arouse their motivation about what they are learning. While what students learn has remained almost the same, the way how they learn it is changing rapidly as technology advances. Holding students´ attention for a longer time is not an easy task. Therefore the use of multimedia in the classroom becomes a great way of getting students engaged. [2] We also intended to encourage the teachers to examine their own beliefs about teaching using video and apply it in a practical way.

## 2. Why Use Video in the Classroom?

The use of videotapes has been a common feature in language teaching for many years. Teachers frequently animate their classses with this material produced for language learning. Using videos in the classroom has many advantages for students. First of all, it helps them learning and remembering different items and processes. They are able to retain more information, understand concepts and feel interested in the particular topic. With video students often make new connections and discover links between various topics and the world around us. [3]

There are many reasons why video can add an extra dimension to the learning experience. We will mention only a few of them [1]:

- Seeing language-in-use

This is one of the main advantages of using video in the class because students do not only hear the sound. They can also see where the conversation takes place, they would be able to observe how the people involved look like, what kind of gestures, facial expression and body language do they use. All these features help them understand the message that is being conveyed, more deeply.

- Cross-cultural awareness

With videos we can bring life to our classrooms. It allows students to look at situations taking place in foreign countries being far away and outside the classroom. It shows a scale of behaviour, customs and habits of particular countries and nationalities and helps students understand the differences.

- The power of creation

Using video cameras gives students the potential to create something memorable. The task of video-making can provoke especially those creative ones to do some interesting pieces of video.

- Motivation

Most students become interested in the topic when appropriate tools are chosen, motivating tasks are used and when they have a chance to see and hear language in use.

Video allows us to:

- take students around the globe, to different English speaking countries
- meet people from the other countries and hear their ideas
- bring literature, plays, music or important scenes from history into the room

By exploiting the medium's power to deliver lasting images, teachers can:

- reach students with a variety of learning styles, especially visual learners, and students with a variety of information acquisition styles
- engage students in problem-solving and investigative activities
- begin to dismantle social stereotypes
- help students practice media literacy and critical viewing skills
- provide a common experience for students to discuss [3]

In case we want to use a video in language teaching, first we have to decide in which part of the lesson would it be suitable to use it and what type of video to choose. When choosing the video we should take into account the age and number of students in our class, their level of English, their interests. We can use video as a warm up activity, listening and speaking activity or it could be intended as a motivation before writing activity.

Basically there are three types of video which can be used in class [1]:

- „Off-air“ programmes

These are programmes recorded from a television channel. We should be aware of suitable length and level of difficulty of the video so as it is not too long or boring and extremely difficult for students to understand. The best pieces are programmes which can be used for a range of activities as speaking, prediction or cross-cultural awareness. While using these be sure you do not break the law because all television programmes have some copyright restrictions.

- „Real-world“ videos

This means separately published video material such as feature films, wildlife documentaries or comedies. However, we have to make a choice about appropriate length and level of the extract, too.

- Language learning videos

These are videos that accompany many of the workbooks published nowadays. They have been designed with students and thus are likely to be comprehensible, closely related to students´ interests and can be used for a number of different activities as well.

## 3. Video Teaching Techniques

The techniques are designed to motivate the students, awaken their curiosity, fit their interests and bring real language into the classroom by the means of dialogues on various topics. We will mention some of them.

- Silent viewing activities (Prediction, Reproduction, Random sound down)
- Sound only activities
- Freeze framing (still picture) activities (Prediction, Reproduction, Using the background, Thoughts and emotions)
- Paired viewing acitivties (Description, Narration, Split class – Description/Narration)

- Role plays [4]

In Silent viewing activities teacher plays the tape without the sound and students have to guess what characters are saying, what kind of music they would use in an extract and why. We can also turn the volume down or mute the sound at random intervals and ask students to fill in missing words.

When doing Sound only activities teacher covers the screen or turns the brightness control down. Students can only hear the sound and listen to a dialogue. They have to guess where it takes place, who the speakers are and what do they look like. Attention can also be paid to music track and other sounds while students have to guess the kind of scene where the story takes place.

We can freeze or stop the picture using Freeze frame or Pause control when practising Freeze framing (still picture) activities. Students have to say what follows or what they think will happen next. They also think of any words or ideas the characters are likely going to say.

In Paired viewing activities one student in each pair turns their back to the screen and the other one faces the screen. Teacher plays the video in a silent mode. The student that is able to see the screen describes it to their partner.

After watching a video students are asked to act out or role play some sequences they have seen before.

## 4. Video – making activities

Video is one of the most powerful forms of communication. We can use it to inform, educate or influence our audience. It aids comprehension, it is one of the most effective ways to gain and hold the attention of learners. Visual learning is also the best of the sensory learning styles - 65% of people say they learn better by seeing or watching. Auditory learning comes in second, with roughly 30% of the population favoring this method. Since video incorporates both senses, it's easy to see how the use of video can be a powerful educational tool that speaks to a full 95% of the population. [6]

Video also teaches students about multimedia communication with action and motion. It can be done within classroom activities which include making a commercial, preparing material and recording news, script writing or editing the video. Its message should be brief, clear and easy to understand.

It helps students reinforce critical thinking, being able to evaluate the problem and find solutions. Other important skills developed by using video are interpersonal communication, public speaking and group discussion making. [5] Interpersonal communication is important mainly in pairwork or groupwork. Students can communicate and cooperate, consider, help each other, divide subtasks and solve problems better. They can analyze problems more properly, make choice from the alternatives and finally make decisions more easily. They also learn how to plan their communication appropriately. They need to think about the structure of the text and the ways how to raise the interest of the audience. These belong to public speaking skill.

There are many ways of making a video. Students can make video simulations, use creative ideas to film some episodes or make videos with a focus on particular language points. Video simulations can provide feedback when students can watch themselves and assess their performance. It also becomes more realistic. Creative ideas expect students to use their imagination and creativity in process of their language learning. Making videos focusing on some grammar topic is the most difficult from all of the above mentioned ways of making it. This task is suitable for more advanced students when they are given one particular grammar topic, they study it carefully and then discuss how to explain grammar to a different group of learners. Finally they do a filmed lecture with some examples of grammar within. [1]

## 5. Conclusion

In our contribution review we wanted to encourage teachers of English language to use video in their lessons. One of the reasons is that visual stimulants in a video generate attention. This is an element which every teacher struggles to meet in their teaching objectives and video can help us to reach the goal. Videos also enhance discussion and can be used like motivation tools for role-plays or any kind of free writing activities.

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# Digitization Workflow at University of Žilina

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**Abstract.** This article aims to give latest information about digitization process at the University of Žilina in the context of Memory of Slovakia – The national center of excellence. We briefly describe hardware and software equipment and process of digitization. Paper describes digital objects created during the process of digitization of cultural heritage ongoing at the University of Žilina and their presentation.

**Keywords:** workflow, digitization, repository, digital objects.

## 1. Introduction

Often digitization is understood as just being the conversion of an analogue information object into a digital format. However, digitization is more than just the technical conversion from analogue to digital. Digitization is a process that involves various stages. It starts with determining digitisation goals, then proceeds with the selection and the preparation of documents, the definition of the quality parameters, the actual conversion from analogue to digital, the quality control of the digital files, the long-term storage, and ends with making the digital content accessible. Hardware and software used for digitization are critical to the success of the whole project [1].

### 1.1. About Memory of Slovakia – The National Centre of Excellence

Memory of Slovakia – The National Centre of Excellence in research, protection and accessibility of cultural and scientific heritage was founded in 2010. This project was developed in cooperation with the Slovak national library in Martin. One of the main objectives of the project was to build up The Centre of Excellence located in the University Library of University of Žilina. It serves as a training centre for students of library and information science.

### 1.2. Hardware

The centre is equipped with modern technology. There are three types of scanner robots.

- **Treventus ScanRobot 2.0 MDS -** is scanner for mass digitization. This automatic scanner is able to scan up 2500 pages per hour. It is used to scan of industrial book production [2].

- **Bookeye 3** - is ideal scanner for digitization of valuable scripts, folders, drawings, plans and especially historical books because this scanner has motorized book cradles, which are gentle on delicate materials and bookbinding [3].

- **XINO S700** - is the fastest automatic scanner, which has been purchased to the Centre of Excellence. The XINO S700 scanning system is equipped with a feeder for 500 sheets. It features batch-oriented processes, professional image editing and standardized user dialogs. The scanner is suitable for single sheets without bookbinding [4].

### 1.3. Software

During the digitization process it is necesary to use different types of softwares for different types of tasks.

- **ScanGate -** is software for image treatment. It has many functions, that are used for automatic correction of scanned pages like automatic border recognition, deskewing, cropping, resizing, binarization, brightness or contrast adaption, unsharp masking, page rotation and many others.

- **MarcEdit -** is a free library metadata software. It includes a built in Z39.50 which is national standard defining a protocol for computer-to-computer information retrieval. It allows user to query other library systems and download bibliographic records [5].

- **ABBYY Recognition Server –** is a server-based OCR software that allows to establish process of converting paper to searchable and reusable electronic documents. ABBYY Recognition Server takes care of the whole document capture routine, providing convenient tools for recognition, verification, attributing, full-text indexing, and document conversion. It converts scanned documents to searchable PDF and PDF/A standard for long-term preservation. Server has 190 supported recognition languages [6].

## 2. Digitization Workflow

The center's digitization workflow describes the standards, specifications, and processes involved in our digitization efforts. Developing a workflow reduced the time to scan and keeping the process consistent [7].

**Stages of workflow:**

**Measurement**

As a first step we create documents containing information about physical conditions of every object before scanning. This involves measurement of paper thickness, book width and height, classification of bookbinding type and type of paper. We define degree of yellowing and degree of damage bookbinding. This document serves for statistical purposes and it helps to researcher to prognose the lifetime of the document. It is a part of submission information package (SIP) which is deposited in archive.

**Scanning**

Choice of scan robot depends on type of documents and its bookbinding. Default resolution is set to 300 DPI. During throughout the process the operator performs continuous quality control.

**Image treatment**

The main steps that may need to be done with batch processing with ScanGate software:

- Rotate the image
- Crop the image
- Adjust tone and color
- Assign or convert color space for the image

- Set the format of the output file

## Metadata

With the MarcEdit software we can download records via z39.50. Z39.50 service offers access to MARC21 bibliographic records from the Slovak national library, the British Library's full catalogue or to a selection of the Library of Congress cataloging records. Metadata are saved as a part of the SIP in XML format.

## Creating Submission Information Package

The result of digitization is packaged in several "delivery packages"- Submission Information Package (SIP). Each SIP will correspond to a particular digitized document. Each SIP will contain all the files that make up a digital image of a document. The preservation master (MST) is the highest-quality digital surrogate of the physical document in TIFF format. As it should accurately represent the original document, this digital copy should not be altered for aesthetic reasons. Web-access copies of documents in JPEG format (TRT) are created for digital exhibits [8].

## Ingesting to the archive

Ingest refers to the processes of preparing data and digital objects for adding to a digital archive and of adding them to the digital archive. In this step master file is stored in archive and presentation copy is sent to ABBYY server for optical character recognition.

## OCR

Optical character recognition is conversion of images into machine-encoded text. ABBYY recognition server is used for OCR. This type of software can automatically analyze images of printed texts and turn it into a form that a computer can process more easily. Recognition of historical fonts poses big problem. The recognitions of Latin-script is still not without mistakes, but results are very satisfying [9].

## Presentation

MediaINFO is a web presentation solution for books, newspapers, manuscripts, maps and other scanned material. It enables visitors to browse, search and use content interactively. Whole interaction is done through Adobe Flash, which is most widespread viewing platform with 99% penetration in browsers. Domain address is http://mi.ceps.mediamatika.sk/

Main application window allows browsing through main categories and detailed browsing through visual tree of subcategories. User can simply (de)select any of the nodes to turn on/off display and searching through specific publications.

Mediainfo has several features of end-user web application:

- full-text search with support for BOOLEAN operators
- filtering and searching through various metadata fields
- the results can be viewed in number of ways (Zoom view, Book view)
- creating personal notes and shared it with other people
- special hyperlinks can be created to link directly to content without required login
- customization (basic changes to backgrounds, transparency level, language)[10]

Fez – another presentation software - is an open source and is actually used for Fedora based digital repository and workflow management system. Domain address is http://fez.ceps.mediamatika.sk/. Structure in a Fedora repository consists of communities (departments of all faculties), every community has collections (researchers, teachers…) and every collection has records (publications). Records can belong to both collections and communities. Its advantage is that it allows presenting the multimedia files.

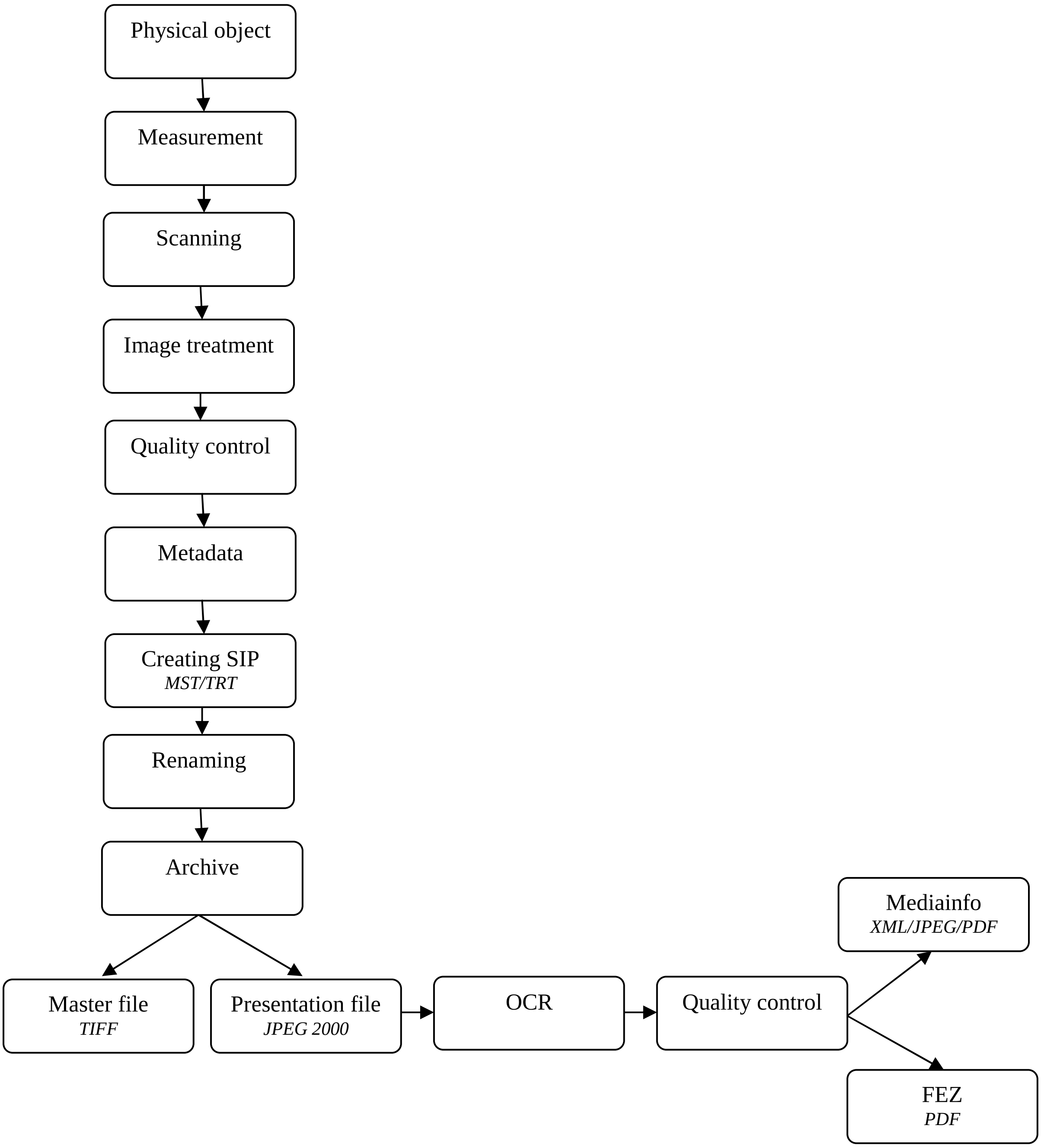


**Fig. 1 Digitization workflow**

## 3. Research in The National Centre of excellence - Memory of Slovakia

Created and modified digital objects form the output of digitization. These digital objects are used for research activities in various field of study, especially by PhD. students.

Currently, The National Centre of Excellence - Memory of Slovakia is working with the research sample, which, was created by scanning documents from the library of The Department of Mediamatics and Cultural Heritage. These objects will serve the students for various purposes and will be accessed by Mediainfo.

**Tab. 1 Digital objects in archive**

| | |
|---|---|
| Objects | 390 |
| Pages | 103 099 |

The second research sample is the book collection from Tranoscius library. Tranoscius is a historical library with rare old books. Books from this collection need very special approach and careful handling. There are 110 objects to primary research of historical books. The processes applied to the sample are: research of optical character recognition for books printed before year 1830 or impact of the bookbinding on the speed and effectivity of a digitization process and the quality of the resulting digital image.

## 4. Conclusion

Digitization is a complex process that affects the number of factors. The aim of the research center is to optimize the digitization process, improve efficiency and speed of the process. Through optimized processes within the archive are fully digitized 103,000 pages which will be shortly available to students, teachers of Mediamatics and Culture Heritage and researchers in the field. Our intention is to create an academic repository as a common space for research staff and academic employees in which they can share their work and to increase the understanding of their research. Creating online library for students and academic repository for the staff increases the level of university.

Centre of Excellence serves not only to simple digitization. The informations about physical condition of the units are recorded. The outputs will be used as a methodology for other digitization projects in the future.

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# Wavelet Neural Networks for Volatility Forecasting. Comparative Analysis with Stochastic Models

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**Abstract.** This paper provides volatility forecasting models by using wavelet analysis, variance analysis and NARX architecture neural networks. Implementation of volatility forecasting is applicable for financial market players: institutional investors (banks, insurance companies, private and state pension funds) as well as private investors for volatility forecasting in financial time series. Developed models are compared with worldwide used stochastic models like GARCH(P,Q), EGARCH(P,Q) and GJR(P,Q). According to research results developed models provide better forecasting results in terms than conditional variance models.

**Keywords:** nonlinear autoregressive with exogenous inputs (NARX) neural networks, wavelet analysis, wavelet neural networks, direct discrete wavelet transform, volatility, historical volatility, implied volatility, volatility index (VIX), variance, the north-east volatility wind' effect, stochastic models, conditional variance models, GARCH(P,Q), EGARCH(P,Q) , GJR(P,Q), time series, forecasting.

## 1. Introduction

Current paper might be applicable for financial market players: institutional investors (banks, insurance companies, private and state pension funds) as well as private investors for volatility forecasting in financial time series.

Volatility is applied for option pricing. Option price is directly related to implied volatility, which is a measure of financial market stability. There are financial instruments, which gain market profit from market instability (price fluctuations). Volatility is also important in asset management strategies. Investors are interested to leave market (or reduce open positions) before the market is ceased by instability. In this case volatility is used as a measure of risk.

There are two types of volatility: **Historical volatility - HV** and **Implied volatility - IV**. Both serve as the measure of scattering, but historical volatility is a function of (asset) past market prices, e.g. German DAX30 stock index, Hang-Seng stock index or stock prices BMW, Volkswagen, Alianz, Adidas, IBM stock prices). The term historical volatility in financial mathematics and econometrics implies measures of scattering - variance and standard deviation.

But implied volatility is a function of option prices. Options are derivative contracts. Historical volatility is not tradable and is used for risk management unlike implied volatility which is tradable; therefore, implied volatility dynamics forecasting has a high practical intended use and volatility is traded, for example, example by using **VIX implied volatility index** trades.

This paper continues the scope of publications about volatility forecasting: [12] and publications about so called north-east volatility wind effect [7-12]. So called north-east volatility wind effect, discovered by author, is keeping following idea: small changes in low frequency signal component volatility leads to significant volatility growth in high-frequency components and consequently to overall volatility growth. [10] Further this idea was implemented in volatility forecasting with wavelet based neural networks in paper [12] (which can be conditionally -

called wavelet neural network models). For this purpose NARX (nonlinear autoregressive with exogenous inputs) [3]. Research is realized with Signal decomposition (wavelet decomposition, described in [4, 15]) with subsequent variance analysis on signal components. For modeling wavelet coefficient variance indicator and absolute changes in trend (low-frequency component) in past are used to forecast historical volatility (it is better to say expected volatility) or implied volatility.

Developed wavelet based neural network models are comparable to stochastic models which are classical used for volatility forecasting: GARCH(P,Q), EGARCH(P,Q) and GJR(P,Q) by institutional investors [17]. According research results developed wavelet neural network models are showing better forecasting performance results from MSE (mean squared error) and R (Pearson correlation) perspectives.

## 2. Volatility Forecasting Problems With Stochastic Models

There are several models for financial time series and volatility forecasting in mathematical statistics and probability theory, e.g. Brownian motion, fractal Brownian motion, conditional variance models (CVM). In CVM family the most popular are generalized autoregressive conditional heteroscedastic - GARCH(P,Q), exponential generalized autoregressive conditional heteroscedastic – EGARCH(P,Q) and Glosten, Jagannathan, Runkle – GJR model.

The key problem of volatility forecasting by using CVM family models in it converges to unconditional variance by increasing forecast horizon [6]. This problem is illustrated in Fig. 1 on GARCH (1,1) example, where by rising conditional variance (in other words forecasted volatility value) is getting towards unconditional variance.

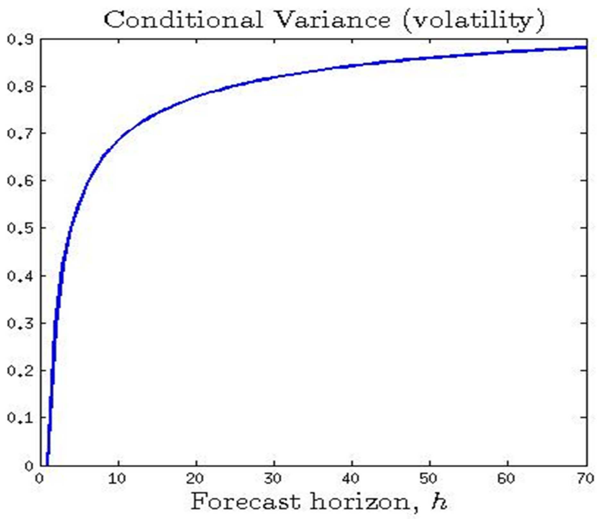


**Fig. 1.** Conditional variance by forecast horizon h for GARCH(1,1) model.

Issue described before subsequently is illustrated in volatility forecasting dynamics on German DAX30 stock index example. The forecast is done by using GARCH (1,1) model for 5 ($h$ = 5) and 12 (h = 12) trading days. For a forecast variable the mean variable of is used, which is compared to empirical values. Volatility forecasting for 5 trading days is shown in Fig. 2a) in time dynamics. Forecast error in time is shown in Fig. 2b) Relevant results shown in Fig. 2A-b) prove that the forecasted volatility is closed to empirical volatility for 5 trading days forecast horizon. Though, rising forecast horizon to dramatically increases forecast error (according the results shown in Fig. 2c-d) results). Analysis results highlight disadvantages of conditional variance model in volatility forecasting. The main reason of the weaknesses is considered in Fig. 1. It is conditional variance (fast or slow) convergence to unconditional variance. This property is typical for all conditional cariance models. Weaknesses and improvement possibilities as well as alternative volatility forecasting models are reviewed in subsequent sections.

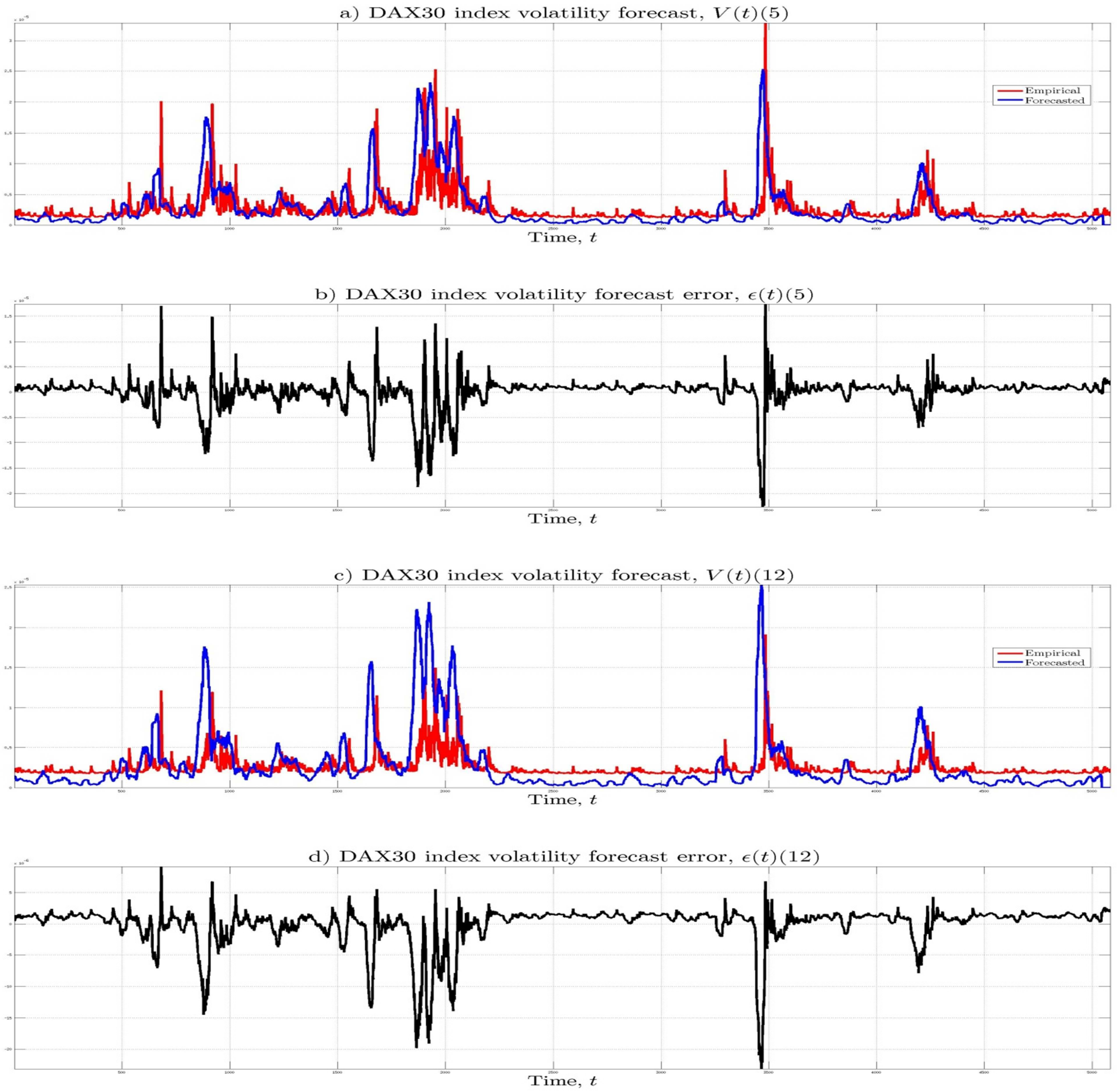


**Fig. 2.** Volatility forecasting at different forecast horizons.

# 3. Wavelet Neural Network Models

## 3.1. Principal Model Components and Functional Dependences

Current section provides analysis of volatility forecasting model development. Developed models use background of north-east volatility wind effect modeling [7, 8, 11] with some simplification of wavelet analysis in order to reduce the number of predictors (factors in models). After discussion and model consideration, the following components of developed models are selected:

wavelet analysis part, variance analysis and neural networks. Model component is shown in Fig. 3.

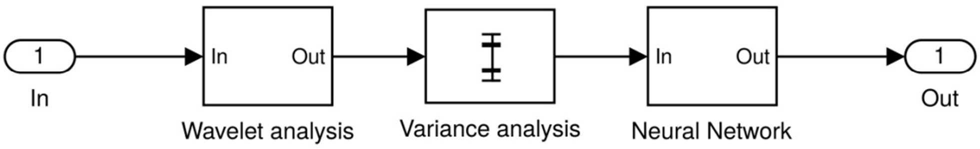


**Fig. 3.** Volatility forecasting model components.

In accordance with the author's experience, the best model for volatility forecasting is nonlinear auto regression implemented in neural network model with NARX (nonlinear autoregressive with exogenous inputs) architecture. The model equation is expressed in equation (1).

$$HV(t+h) = \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{HV}(d,hn)HV(t-d)\big) + \\ + \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{dSa}(d,hn)dSa(t-d)\big) + \\ + \sum_{d=1}^{D}\sum_{hn=1}^{H} F\big(W_{V}(d,hn)V(t-d)\big) + b. \quad (1)$$

Nonlinear dependencies are described by sigmoidal function

$$F(u) = \frac{1}{1-e^{-u}}.$$

Formula (1) shows that each argument $HV, dSa, V$ is related to forecasted volatility $HV(t+h)$ by weight matrices. Each weight matrix has lag parameter $D$ and hidden neuron number parameter $H$. The model work is affected with parameters $H$ and $D$ which are related to neural network architecture. Neural network architecture can be optimized by using discrete optimization in genetic algorithm way or by using enumeration approach [5]. Modeling of algorithm is considering in details in other papers of the author.

### 3.2. Model Performance Results

This subsection contains modeling results. The relevant research results, worked out via wavelet neural network models show high performance results in volatility forecasting. For comparative analysis two indicators were selected for outputs and targets data performance: Pearson correlation (R) and mean squared error (MSE). For analysis several stock index data were selected.

Comparative analysis includes German DAX30 index, American SP500 index, Japan NIKKEI225 and China Hang-Seng indexes. Forecast results are compared for developed (author) models and best stochastic model for certain index. Stochastic models are selected from range: GARCH(1-3,1-3), EGARCH(1-3,1-3), GJR(1-3,1-3). The forecast is done for more than 70 trading days horizon. The results are compared on independent set (test and validation set) and on complete set. Results are illustrated graphically in Fig. 4.

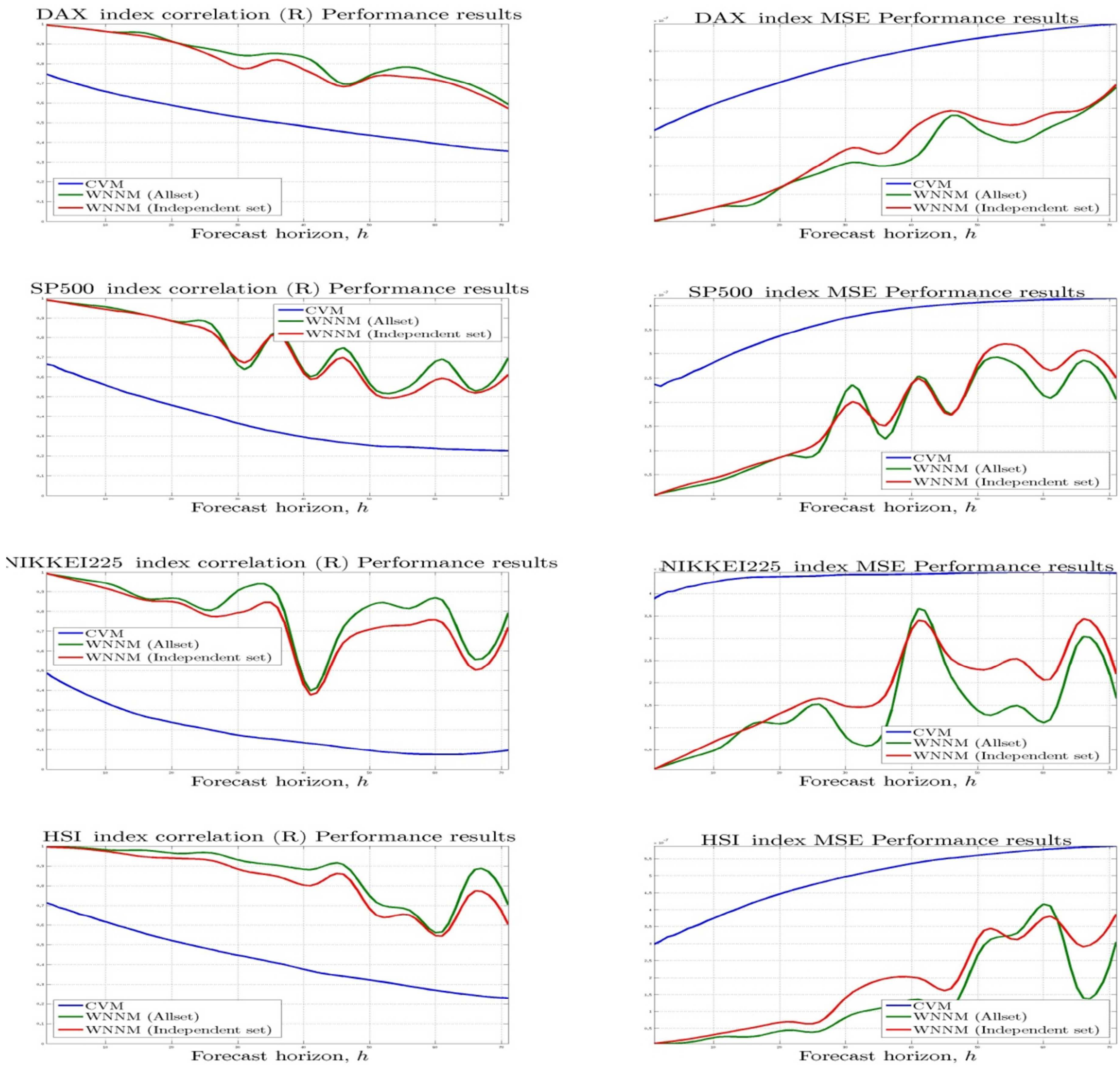


**Fig. 4.** Forecast results comparative analysis.

# 4. Conclusion

Investigating volatility forecasting results for DAX30 stock index, the author concludes the dynamics of worked out models in terms of closely approximate volatility, forecast error. Comparing acquired results to stochastic model results author concludes that developed models approximate volatility dynamics more closely than stochastic models. Hereinafter, model comparative analysis is done.

Comparative analysis includes German DAX30 index, American SP500 index, Japan NIKKEI225 and China Hang-Seng indexes. Forecast results are compared for developed (author) models and best stochastic model for certain index. Stochastic models are selected from range: GARCH(1-3,1-3), EGARCH(1-3,1-3), GJR(1-3,1-3). The forecast is done for more than 70 trading days horizon. The results are compared on independent set (test and validation set) and on complete set.

Comparing the results, the author concludes that developed models show higher performance results than stochastic models (GARCH, EGARCH, GJR) for all forecast horizons. Conclusions are binding for all analyzed stock indexes - Hang-Seng, NIKKEI225, SP500, DAX30. Developed models show better results in all sets (including independent set). From both - correlation R and MSE (mean squared error) perspectives.

Consequently, based upon the relevant research results and additional tests done from neural network performance perspective, the author concludes that there is small difference in independent set and full set results for short-term forecasts (up to 21-26 trading days, depending on index). This gives grounds to believe that the observed horizon results are stable and safe operability in volatility forecasting.

Longer term horizon use of the developed models is possible, though it is recommended to be more selective in choice of the model which, in its turn, increases overall uptime indicator. Notwithstanding, author models provide better results than stochastic models. The above statement is illustrated in Fig. 4 model results.

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# Intertextuality in Language and Literary Education

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**Abstract.** This paper demonstrates some theoretically postulated ideas of using intertextuality as a key concept in the process of gaining linguistic competence as well as a set of potential innovative techniques which might be applicable at school language learning and literary education. The constitution of a dialogue between literary text and a reader can be viewed as a functional interconnection of reproductive and productive activities during the school lessons. Thus active participation of students in this process by means of critical reading and creative writing, integrated as two component parts of language studies, can be judged as a vital strategy and subsequently implemented into educational programmes.

**Keywords:** Intertextuality. Dialogism. Textual Manipulations. Creation of Meaning. Creative Writing.

## 1. Introduction. General Notes on Intertextuality as a Starting Point

In this article we will promote the idea of utilising intertextuality in the classroom in the form of creative writing exercises. Lots of attempts to define this phenomenon have been made yet the cases of using it without explicit naming are even more frequent. The most general definition is that we deal with the interrelation between texts, especially works of literature, and the way that similar or related texts influence, reflect, or react to each other.

Intertextuality as the shaping of a text's meaning by another text is reflected on various levels of social practice and its presence is nowadays considered as self-evident. The most frequent cases of intertextual operations are duplication (a sequence of words that occur in two texts such as in quotation), naming and reference (as occurs in citations), proximal association (as occurs among related chapters in edited books) and sequential association (an established sequence of related texts such as a reply to a letter or e-mail). But besides these explicit forms it can be implied through a variety of literary devices (e.g., allusion, metonymy, synecdoche) and certain stylistic means (repetition of a stress, sound, or rhyme pattern across two or more texts) [1].

We can mention also some other common intertextual figures: allusion, quotation, calque, plagiarism, translation, pastiche and parody.

## 2. The Role of Intertextual Operations in Educational Practice

Since Julia Kristeva revealed that reading and writing are tied together in one continuum, language teachers had to examine this new quality of literary communication at school level. It is still being experienced in new variations provided by creative teachers to enhance learners´ basic language acquisition skills as well as a set of supplementary competences in addition to the primary linguistic competence (cultural, social, communicative competence etc.) Kristeva broke with traditional notions of the author's "influences" and the texts´ "sources". A literary work, then, is not simply the product of a single author, but of its relationship to other texts and to the structures of language itself. "Any text," she argues, "is constructed of a mosaic of quotations [2].

The mutual correspondence which we have mentioned is crucial for this concept, as it allows the interchange of thoughts and emotions. This would not be thinkable without continuing interaction between the two counterparts of this communication act. In school practice just this is the feature we can benefit from reciprocal nature of intertextuality establishing a unique kind of exchange between the two participants of various textual, intratextual and intertextual operations. Due to its variable character we can redefine reading and writing into one reversible process instead of two isolated activities (as usually performed at school language lessons).

The two sides of communication are bound together through the medium, displaying certain message of the text which should be interpreted. No more is the reader determined to the passive role in the act of re-creation of the meaning. Submission would not work in such circumstances where creative flow of energy can be applied. We should even talk about the reference writer-reader as a valid contract which can be versatile. But the relationship of the two subjects taking part in communication is specified by existence of its once fixed evidence, which can be granted to the literary text. This is why the role of a text is unique here; as we dare say that construction of the meaning is constantly happening. It is still in process which is based on its current exploration by a creative being, it always remains undone yet re-produced over and over. Here comes the intertextual viewpoint supporting the concept that the meaning of a text does not reside in the text. Meaning is produced by the reader in relation not only to the particular text, but also to the complex network of texts invoked in the reading process.

Let´s see what Frank Smith claims about the participants of this kind of intertextual contract: “There are three parties to every transaction that written language makes possible: a writer, a reader, and a text. And of the three, the text is the pivot. Although texts may be (and often are) studied independently of the other two, neither writers nor readers can exist without a text. Writers must produce texts and readers must interpret them, and the text always stands between the two, a barrier as well as a bridge. Writers cannot reach through a text to the reader beyond, any more than a reader can penetrate the text to make direct contact with the writer. Like a river that permits communication between one shore and another, the text is also an obstacle that keeps the two sides apart” [2].

A good reason to implement intertextual acts into school education is the cognitive aspect of textual communication (together with potential options of meta-cognition) as well as self-reflection: “Not only can a piece of writing communicate thought from writer to reader, but also the act of writing can tell the author things that were not known (or not known to be known) before the writing began” [2].

## 3. On Some of Practical Implications of Intertextual Acts. Dialogism.

As Bloom and Bailey claim, “exploration of intertextuality also provides a different perspective or definition of education. If part of what people do in events is use intertextuality to create meaning, to construct a cultural ideology, to establish history, then education can be viewed as acquisition of the communicative competence to participate in the event and in subsequent events in an appropriate historical manner. This is an overly complex way of saying that participation in one event leads to participation in another, and that people, while they are the creators of events, are also caught up in them” [3].

But what is exceptionally significant in the educational context, is the dialogical nature of intertextually based communication acts. Here Bakhtin situates the text within history and society, which are then seen as texts read by the writer, and into which he inserts himself by rewriting them. Diachrony is transformed into synchrony, and in light of this transformation, linear history appears as abstraction.

This is what makes it turn to a dialogical quality of such communication. Bakhtin notes three dimensions or coordinates of a dialogue, such as writing subject, addressee, and exterior texts. Any text is constructed as a mosaic of quotations; any text is the absorption and transformation of another. The notion of intertextuality replaces that of intersubjectivity. The word is “spatialized; through the notion of status, it functions in three dimensions (subject-addressee-context) as a set of dialogical, semic elements or as a set of ambivalent elements” [4].

Dialogism is inherent in language itself in various oppositions that operate in speech performance. This results in the implication that the minimal unit of poetic language is at least double, in terms of one and other. Bakhtin's term dialogism thus implies the double: language, and another logic: the logic of distance and relationship between the different units of a sentence or narrative structure [5].

Recognition of specific concept of intertextual transition can play decisive role at various stages of language education, especially in accessing linguistic and cultural competence as provided by its dialogical structure when incorporated into curricular social practices at school. Its significance is due to the concept of dialogization which helps students and teachers take reciprocal parts on the literary communication. Supposing that such individualities as young students constantly need to make sense of the world around, especially at the level of abstract cognition, the idea of passing on as much learning responsibility as possible to them is likely to be highly appreciated.

In the field of language and literary education this can be done by implementation of the intertextual dialogue with primary texts, but it can proceed to further storeys (to communicate at further levels of interaction). This moment of intertextually oriented dialogue is just what we find very productive. In the classroom circumstances it can be realized by special set of reproductive and productive tasks based on various types of textual manipulation. This primary transaction can result into subsequent intertextually based techniques which combine both interpretative and creative mental strategies. A versatile method which we suggest as notably appropriate for this purpose is creative writing. Thus the functional interconnection of opposite processes of critical reading (together with textual analysis, critical thinking, critical philosophizing, role-playing etc.) and wide range of techniques of creative writing can be adjusted for specific needs of school language and literary education.

## 4. Conclusion

Intertextuality can be viewed as a function of social practices associated with the use of language. The qualitative research of applied intertextually-based techniques has been done in 2010 as a part of our dissertation [6]. The hypothesis about significance of introduced innovative approach have been proven valid, especially the findings about productivity of dialogism in the process of textual operations, conversion of reading into re-writing act, benefiting from developed interpretational skills in the area of productive creating of the meaning and broadening the space where individual capacity can be placed (interpersonal level). All these processes helped students do progress in their individual formation.

We have announced the implicit presence of intertextual processes in designed exercises of creative writing by practical realization of lessons using alternative methods of process-oriented activities. We have focused on divergent tasks as they enhance students´ creative effort and motivate them in searching for original expression. In this process there is a connection of cognitive features in learners´ development by means of acknowledgment of invariant structures and mechanisms of literary text with following application in the way of manipulation with the textual strategies. The tasks were constructed due to respect the characteristics of intertextual transformation in conditions of school education. As a sample of guidelines to the creative exercises we have introduced a set of particular tasks based on with re-writing of texts by well-known authors (e.g. Voyelles by J. A. Rimbaud featuring inspiring activities with associations of colored thinking). We have to say that all of these conclusions refer to demands for implementation of suggested methods into school documents in order to their exploitation in educational practice.

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# Sufficient Conditions for the Existence of Global Solutions of a Linear Scalar Equation

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**Abstract.** This paper aims to find the sufficient conditions for the existence of global solutions of the linear scalar equation with multiple delays. Sufficient conditions for linear scalar equation with one delay are modified for equation with multiple delays. Transcendental equations are used. An example illustrates the result.

**Keywords:** delayed differential equation, global solution, sufficient conditions.

## 1. Introduction

Let us consider the equation

$$\dot{y}(t) = -\sum_{i=1}^{m} c_i(t) y\left(t - \tau_i(t)\right). \tag{1}$$

Throughout the paper we will assume that $c_i\colon \mathbb{R} \to (0, \infty)$ are Lipschitz continuous and bounded functions on every interval $(-\infty, \theta]$, for $\theta \in \mathbb{R}$, $i = 1,\ldots, m$ and the delays $\tau_i\colon \mathbb{R} \to (0, r]$ are Lipschitz continuous functions on every interval $(-\infty, \theta]$, for $\theta \in \mathbb{R}$, $i = 1,\ldots, m$ and $r$ is a positive constant. The symbol "˙" denotes a derivative, which we assume is at least right-sided.
A global solution of the equation (1) is defined as a continuously differentiable function $y\colon \mathbb{R} \to \mathbb{R}$ satisfying (1) on $\mathbb{R}$.

This paper is organized as follows. In section 2 auxiliary inequalities and criteria for existence of global solution of retarded functional differential equations are cited. The result for equation of a type (1) and an example are given in section 3. In the last section the conclusion is formulated.

## 2. Auxiliary Inequalities and Lemma

Let us consider a retarded functional differential equation

$$\dot{y}(t) = f(t, y_t), \tag{2}$$

where $f : \Omega \to \mathbb{R}$, $\Omega \subseteq \mathbb{R} \times C_r$ is a continuous quasi-bounded map which satisfies a local Lipschitz condition with respect to the second argument. $C_r$ is the Banach space of the continuous functions from $[-r, 0]$ into $\mathbb{R}$. Below we assume (we do not give concrete specification of $\Omega$) that the used operations concerning $f$ are well defined.

Let $\lambda : (-\infty, \theta) \to \mathbb{R}$ be a continuous function where $\theta \in \mathbb{R}$, $t^* \in (-\infty, \theta]$ and $k \in (0, \infty)$. Define

$$I_{(-\infty,\theta]}\left(k, t^*, \lambda\right) = k.\exp\left(-\int_{t^*}^{t} \lambda(s)\, ds\right), \; t \in (-\infty, \theta],$$

and

$$\left(T_{(-\infty,\theta]}\lambda\right)(t) = -k^{-1}.\exp\left(\int_{t^*}^{t} \lambda(s)\, ds\right).f\left(t, \left(I_{(-\infty,\theta]}\left(k, t^*, \lambda\right)\right)_t\right), \; t \in (-\infty, \theta].$$

The following lemma is a special case of Theorem 2.1 from [1]:

**Lemma 1.** Let $t^* \in \mathbb{R}$, $\theta \in \mathbb{R}$ be fixed and $\theta \geq t^*$. Moreover we assume:

(i) For any $M \geq 0$ there exists a constant $K$, such that for all $t$, $t' \in (-\infty, \theta]$ and for any continuous function $\lambda : (-\infty, 0) \to \mathbb{R}$ with $|\lambda| \leq M$

$$\left| \left(T_{(-\infty,\theta]}\lambda\right)(t) - \left(T_{(-\infty,\theta]}\lambda\right)(t')\right| \leq K|t - t'| . \qquad (3)$$

(ii) There are $k \in (0, \infty)$ and continuous functions $\lambda_i : \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, which are bounded on every interval $(-\infty, \theta_1]$, for $\theta_1 \in \mathbb{R}$ and which satisfy

$$\lambda_1(t) \leq \lambda_2(t),$$

$$\lambda_1(t) \leq -k^{-1}.\exp\left(\int_{t^*}^{t} \lambda_1(s)\, ds\right). f\left(t, \left(I_{(-\infty,\infty)}\left(k, t^*, \lambda_1\right)\right)_t\right),$$

$$\lambda_2(t) \geq -k^{-1}.\exp\left(\int_{t^*}^{t} \lambda_2(s)\, ds\right). f\left(t, \left(I_{(-\infty,\infty)}\left(k, t^*, \lambda_2\right)\right)_t\right)$$

on $\mathbb{R}$.

(iii) For any continuous functions $\Lambda_i: \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, such that $\Lambda_1(t) \leq \Lambda_2(t)$, $t \in \mathbb{R}$ we have $T_{(-\infty,\infty)}\Lambda_1 \leq T_{(-\infty,\infty)}\Lambda_2$.

Then there exists a global solution $y: \mathbb{R} \to \mathbb{R}$ of (2) satisfying $y(t^*) = k$ and

$$k.\exp\left(-\int_{t^*}^{t} \lambda_1(s)\, ds\right) \leq y(t) \leq k.\exp\left(-\int_{t^*}^{t} \lambda_2(s)\, ds\right) \text{ for } t < t^*, \qquad (4)$$

$$k.\exp\left(-\int_{t^*}^{t} \lambda_2(s)\, ds\right) \leq y(t) \leq k.\exp\left(-\int_{t^*}^{t} \lambda_1(s)\, ds\right) \text{ for } t > t^*. \qquad (5)$$

## 3. Sufficient Conditions for Existence of Global Solutions of (1)

Below we apply Lemma 1 to equation (1). As a result we get not only the existence of global solutions of equation (1) but also their upper and lower bounds on $\mathbb{R}$.

**Theorem 1.** Suppose that there are continuous functions $\lambda_1$, $\lambda_2$: $\mathbb{R} \to \mathbb{R}$, satisfying the inequalities

$$\lambda_1(t) \leq \lambda_2(t),$$

$$\lambda_1(t) \leq \sum_{i=1}^{m} c_i(t) \exp\left(\int_{t-\tau_i(t)}^{t} \lambda_1(s)\, ds\right), \qquad (6)$$

$$\lambda_2(t) \geq \sum_{i=1}^{m} c_i(t) \exp\left(\int_{t-\tau_i(t)}^{t} \lambda_2(s)\, ds\right) \qquad (7)$$

on $\mathbb{R}$ and being bounded on every interval of the form $(-\infty, \theta]$ for $\theta \in \mathbb{R}$. Then there exists a global solution $y$ of (1) such that $y(t^*) = 1$, moreover

$$\exp\left(-\int_{t*}^{t} \lambda_1(s)\, ds\right) \leq y(t) \leq \exp\left(-\int_{t*}^{t} \lambda_2(s)\, ds\right) \text{ if } t < t^*, \qquad (8)$$

$$\exp\left(-\int_{t*}^{t}\lambda_2(s)\,ds\right) \le y(t) \le \exp\left(-\int_{t*}^{t}\lambda_1(s)\,ds\right) \text{ if } t > t^*. \tag{9}$$

**Proof.** We are going to apply Lemma 1 to the equation (1), where $f(t, y_t) := -\sum_{i=1}^{m} c_i(t) y\big(t-\tau_i(t)\big)$. Then we get

$$(T_{(-\infty,\theta]}\lambda)(t) := \sum_{i=1}^{m} c_i(t) \exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right),\ t \in (-\infty, \theta]. \tag{10}$$

Let us verify the inequality (3). For $t$, $t' \le \theta$ the following is true

$$\left|(T_{(-\infty,\theta]}\lambda)(t) - (T_{(-\infty,\theta]}\lambda)(t')\right| = \left|\sum_{i=1}^{m} c_i(t)\exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) - \sum_{i=1}^{m} c_i(t')\exp\left(\int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds\right)\right|$$

$$\le \sum_{i=1}^{m}\left|c_i(t)-c_i(t')\right|\exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) + \sum_{i=1}^{m} c_i(t')\left|\exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) - \exp\left(\int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds\right)\right|.$$

From the assumptions stated in the Introduction, coefficients $c_i$ and the delays $\tau_i$, $i = 1,\dots, m$, are Lipschitz continuous and bounded functions. Therefore for each $i$ there exist positive constants $M_{ci,\theta}$, $L_{ci,\theta}$ and $L_{\tau i,\theta}$ such that

$$c_i(t) \le M_{ci,\theta},\ |c_i(t) - c_i(t')| \le L_{ci,\theta}\,.\,|t - t'|$$

and

$$|\tau_i(t) - \tau_i(t')| \le L_{\tau i,\theta}\,.\,|t - t'| \text{ for every } t, t' \le \theta.$$

By Lagrange's mean value theorem there exist constants $K_i$ between $\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds$ and $\int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds$, such that

$$\exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) - \exp\left(\int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds\right) = e^{K_i}\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds - \int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds\right).$$

Moreover λ and $\tau_i$ are bounded on the interval $(-\infty, \theta]$, so the following is true $\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) \le Mr$ and therefore $\exp\left(\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds\right) \le e^{Mr}$ and also $e^{K_i} \le e^{Mr}$. Finally,

$$\left|\int_{t-\tau_i(t)}^{t}\lambda\,(s)\,ds - \int_{t'-\tau_i(t')}^{t'}\lambda\,(s)\,ds\right| = \left|\int_{t'}^{t}\lambda\,(s)\,ds + \int_{t-\tau_i(t)}^{t'-\tau_i(t')}\lambda\,(s)\,ds\right| \le M|t-t'| + M\left|t'-\tau_i(t') - (t-\tau_i(t))\right|$$

$$\le 2M|t-t'| + ML_{\tau i,\theta}|t-t'|.$$

To finish the estimation let us summarize all the facts. We have

$$\left|(T_{(-\infty,\theta]}\lambda)(t) - (T_{(-\infty,\theta]}\lambda)(t')\right| \le \sum_{i=1}^{m} L_{ci,\theta}|t-t'|e^{Mr} + \sum_{i=1}^{m} M_{ci,\theta}e^{Mr}\left[2M|t-t'| + ML_{\tau i,\theta}|t-t'|\right]$$

$$= e^{Mr}\left[\sum_{i=1}^{m} L_{ci,\theta} + \sum_{i=1}^{m} M_{ci,\theta}\left(2M + ML_{\tau i,\theta}\right)\right]|t-t'|$$

Let $M_{c,\theta} = \max_{1\le i\le m}\{M_{ci,\theta}\}, L_{c,\theta} = \max_{1\le i\le m}\{L_{ci,\theta}\}, L_{\tau,\theta} = \max_{1\le i\le m}\{L_{\tau i,\theta}\}$. Then we can write

$$\left|(T_{(-\infty,\theta]}\lambda)(t)-(T_{(-\infty,\theta]}\lambda)(t')\right|\le e^{Mr}\left[mL_{c,\theta}+mM_{c,\theta}\left(2M+ML_{\tau,\theta}\right)\right]\left|t-t'\right|,$$

which implies that the condition (i) of Lemma 1 is valid for $K = m.\exp(Mr)[L_{c,\theta} + M_{c,\theta}(2M+ML_{\tau,\theta})]$. Condition (ii) follows from the inequalities (6), (7) for $k = 1$. Finally, condition (iii) follows directly from the definition of T given by the relation (10). Therefore, in accordance with Lemma 1, there exists a global solution $y$ of (1) such that $y(t^*) = 1$ and inequalities (8) and (9) follow from (4) and (5).

Finding the functions $\lambda_1$ and $\lambda_2$ is not an easy task and there is no general recipe how to search them. In the following part we will describe the procedure how to define such functions for certain type of equations. We will need next lemma (see [2, Lemma 1]).

**Lemma 2.** Let $a > 0$, $\tau > 0$ and $ae\tau < 1$. Then the equation $\lambda = a\,.\,\exp(\lambda\tau)$ has just two different positive roots $\lambda^*$, $\lambda^{**}$ and $\lambda^* < 1/\tau < \lambda^{**}$.

**Theorem 2.** Let $t^*$ be fixed. Moreover we assume that functions $c_i(t)$, $i = 1,\dots, m$ satisfy $0 < c \le c_i(t) \le C$, $t\in\mathbb{R}$, where $c$, $C$ are constants and the delays $\tau_i(t)$, $i = 1,\dots, m$ satisfy $0 < \delta \le \tau_i(t) \le r$, $t\in\mathbb{R}$, where $\delta$, $r$ are constants. If $C < 1/(rme)$, then there exists a global solution $y$ of (1) satisfying $y(t^*) = 1$ and the inequalities

$$\exp\left(-\lambda^*(t-t^*)\right)\le y(t)\le \exp\left(-\lambda^{**}(t-t^*)\right) \text{ if } t < t^*, \tag{11}$$

$$\exp\left(-\lambda^{**}(t-t^*)\right)\le y(t)\le \exp\left(-\lambda^*(t-t^*)\right) \text{ if } t > t^*, \tag{12}$$

where $\lambda^*$ is the smaller root of transcendental equation $\lambda = mc.\exp(\lambda\delta)$ and $\lambda^{**}$ is the bigger root of $\lambda = mC.\exp(\lambda r)$.

**Proof.** According to Lemma 2, the transcendental equation $\lambda = mc.\exp(\lambda\delta)$ has a positive root $\lambda^* < 1/\delta$ and similarly, the equation $\lambda = mC.\exp(\lambda r)$ has a root $\lambda^{**} > 1/r$. Let us set $\lambda_1(t) = \lambda^*$ and $\lambda_2(t) = \lambda^{**}$. We will show, that these functions suit the assumptions of Theorem 1.

Let us define an auxiliary function $f(\lambda) = mC.\exp(\lambda r)-mc.\exp(\lambda\delta)$. The derivative of the function is following

$$f'(\lambda) = mCr\mathrm{e}^{\lambda r} - mc\delta\mathrm{e}^{\lambda\delta} = mCr\mathrm{e}^{\lambda\delta}\left(e^{\lambda(r-\delta)}-\frac{c\delta}{Cr}\right).$$

We assume that $r \ge \delta$ and $C \ge c$, therefore $\exp(\lambda(r-\delta)) \ge 1$ and $(c\delta)/(Cr) \le 1$. So the derivative is non-negative and function $f(\lambda)$ is non-decreasing. Moreover $f(0) \ge 0$, $f(\infty) = \infty$, hence we can conclude that the following inequality holds

$$mC\mathrm{e}^{\lambda r} \ge mc\mathrm{e}^{\lambda\delta} \text{ if } \lambda > 0. \tag{13}$$

Because of the inequality (13), it is true that

$$\lambda^* \le \lambda^{**}. \tag{14}$$

Since the coefficients $c_i(t)$ and time delays $\tau_i(t)$ are bounded, we can estimate the following sum

$$\sum_{i=1}^{m} c_i(t)\exp\left(\int_{t-\tau_i(t)}^{t}\lambda^* ds\right) = \sum_{i=1}^{m} c_i(t)\exp\left(\lambda^*\tau_i(t)\right) \ge mc.\exp\left(\lambda^*\delta\right) = \lambda^*, \tag{15}$$

and similarly we get

$$\sum_{i=1}^{m} c_i(t)\exp\left(\int_{t-\tau_i(t)}^{t}\lambda^{**} ds\right) \le mC.\exp\left(\lambda^{**}r\right) = \lambda^{**}. \tag{16}$$

By Theorem 1 (the assumptions hold due to inequalities (14)-(16)) there exists a global solution of (1) satisfying $y(t^*) = 1$ and the inequalities (11) and (12).

**Example.** Consider the equation

$$\dot{y}(t) = -\frac{1+0{,}5\sin t}{35\mathrm{e}}\, y(t-1) - \frac{1+0{,}5\cos t}{30\mathrm{e}}\, y(t-5) \qquad (17)$$

on (-∞, ∞). This is an equation of type (1) with $m = 2$, $c_1(t)$ = (1+0,5sin $t$)/(35e), $c_2(t)$ = (1+0,5cos $t$)/(30e), $\tau_1(t) = 1$ and $\tau_2(t) = 5$. Let us set $c$ = 1/(70e), $C$ = 1/(20e), δ = 1 and r = 5. Obviously, $C$ = 1/(20e) < 1/(10e) = 1/($rm$e) and, according to Theorem 2, there exists a global solution $y(t)$ of (17) satisfying $y(t^*) = 1$ and inequalities (11) and (12).
By the Wolfram Alpha software, available online on webpage [3], we can verify that transcendental equation $\lambda$ = 2/(70e).$\mathrm{e}^{\lambda}$ has a root $\lambda^* \cong 0{,}0106$ and $\lambda$ = 2/(20e).$\mathrm{e}^{5\lambda}$ has a root $\lambda^{**} \cong 0{,}5357$. Let us set $\lambda_1(t) = 0{,}01$ and $\lambda_2(t) = 0{,}5$. It is true, that $\lambda_1(t) \le \lambda_2(t)$ for $t \in \mathbb{R}$. Moreover

$$\frac{1+0{,}5\sin t}{35\mathrm{e}}\exp\left(\int_{t-1}^{t} 0{,}01\, ds\right) + \frac{1+0{,}5\cos t}{30\mathrm{e}}\exp\left(\int_{t-5}^{t} 0{,}01\, ds\right) \ge \frac{2}{70\mathrm{e}} e^{0{,}01} \cong 0{,}0106 \ge 0{,}01 = \lambda_1(t)$$ and for

$\lambda_2(t)$ the following inequality holds

$$\frac{1+0{,}5\sin t}{35\mathrm{e}}\exp\left(\int_{t-1}^{t} 0{,}5\, ds\right) + \frac{1+0{,}5\cos t}{30\mathrm{e}}\exp\left(\int_{t-5}^{t} 0{,}5\, ds\right) \le \frac{2}{20e}\mathrm{e}^{2{,}5} \cong 0{,}4482 \le 0{,}5 = \lambda_2(t)\,.$$

We have found functions $\lambda_1(t)$ and $\lambda_2(t)$, such that they fulfill all the conditions stated in Theorem 1. If we set $t^* = 0$, the solution $y(t)$ satisfies the inequalities

$$\exp\left(-0{,}01t\right) \le y(t) \le \exp\left(-0{,}5t\right) \text{ for } t < 0,$$

$$\exp\left(-0{,}5t\right) \le y(t) \le \exp\left(-0{,}01t\right) \text{ for } t > 0.$$

## 4. Conclusion

In this paper we have given the sufficient conditions for existence of global solutions of equation (1). One of the conditions is the existence of special continuous functions described above. For equations with bounded coefficients and time delays also the upper and lower bounds for searched functions are stated.

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# The Usage of Trefftz Functions and Homotopy Perturbation Method for Solving Nonlinear Stationary Heat Equation

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**Abstract.** In the paper homotopy perturbation method combined with the Trefftz functions is used to solve nonlinear stationary heat conduction problems. The two most important steps in application of the homotopy perturbation method are to construct a suitable homotopy equation and to choose a suitable initial guess. The homotopy equation should be constructed such that when the homotopy parameter is zero, it can approximately describe the solution property, and the initial solution can be chosen with an unknown parameter, which is determined after one or two iterations. In each step the exact solution is approximated directly by linear combination of the Trefftz functions and particular solution of nonhomogeneous part of the equation. The presented example shows the usefulness of the method. The results reveal that the method is effective and simple. The obtained errors were compared with those obtained by Picard's iteration method.

**Keywords:** Homotopy perturbation method, Trefftz function, nonlinear stationary heat conduction.

## 1. Introduction

In the recent years, with the rapid development of nonlinear science, the development of numerical techniques for solving nonlinear equations is a subject of considerable interest. Some investigators also proposed the combination of two methods for obtaining the approximate solution to nonlinear partial differential equations. In this paper it is suggested the combination of the homotopy perturbation method and Trefftz functions method.

The fundamentals of the Trefftz method were presented by Erich Trefftz in 1926 in the paper [14]. Next, a lot of scientists developed this theory, including Herrera and Sabina, Jirousek et al. and Kupradze [8, 9, 11]. Also comprehensive monographs exist concerning the Trefftz functions method [1, 4, 10, 12, 13]. In general the method is addressed to solving linear differential equation. The solution is approximated by a linear combination of the functions which satisfy the given equation identically. Minimizing the error describing fulfilling of the approximate solution to the boundary conditions leads to determination of the unknown coefficients of the linear combination.

The homotopy perturbation method was introduced by Ji-Huan He [6, 7], for solving differential and integral equations, linear and nonlinear. It is the subject of extensive analytical and numerical studies. In this method the solution is considered as the summation of an infinite series which converges rapidly to the exact solutions. The homotopy perturbation method has been shown to solve a large class of nonlinear differential problems effectively, easily, and accurately. This method has found application in different fields of nonlinear equations such as fluid mechanics and heat transfer [3]. This method is also adopted for solving the pure strong nonlinear second-order differential equations [2].

The aim of this paper is to apply the homotopy perturbation method combined with Trefftz functions to nonlinear stationary heat equation. The obtained results confirm the power, simplicity and easiness of the method to implement.

## 2. Presentation of the Homotopy Perturbation Method

Let us consider the nonlinear equation:

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{1}$$

with its corresponding boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \partial\Omega, \tag{2}$$

where $A$ is a nonlinear partial differential operator, $B$ is a boundary operator, $f(r)$ is a known analytical function, $u$ is an unknown function, $\Omega$ is a bounded subset of $R^n$ and $\partial\Omega$ is a boundary of $\Omega$. Let us assume that nonlinear operator $A$ can be decomposed in accordance with the formula:

$$A = L + N,$$

where $L$ and $N$ are the linear and nonlinear parts of $A$, respectively. Therefore, (1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0. \tag{3}$$

To solve (3) we construct a homotopy $v(r, p)$:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \quad p \in [0,1], \tag{4}$$

where $p$ is an embedding parameter and $u_0$ is an initial approximation of (1), which satisfies the boundary conditions (2). In this method, $v(r, p)$ can be expanded into a power series with respect to the parameter $p$:

$$v(r, p) = v_0 + pv_1 + p^2v_2 + p^3v_3 + \cdots. \tag{5}$$

The solution of (1) can be approximated by setting $p=1$ in (5):

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + v_3 + \cdots. \tag{6}$$

Generally, the series (6) is convergent and its convergent rate depends on the nonlinear operator $A(v)$, that has been proved by J.H. He in [6, 7].

By choosing an initial approximation $u_0$, substituting (5) into (4) and comparing coefficients of the terms with the identical powers of $p$, lead to:

$$p^0: \quad L(v_0) - L(u_0) = 0,$$

$$p^1: \quad L(v_1) + L(u_0) + N(v_0) - f(r) = 0,$$

$$p^2: \quad L(v_2) + N(v_1) = 0,$$

$$\dots$$

$$p^n: \quad L(v_n) + N(v_{n-1}) = 0.$$

Equations resulting from $p^n$, $n = 1,2,\ldots$ are linear and inhomogeneous. If the Trefftz functions for operator $L$ are known then the solution can be approximated by:

$$v = \sum_{n=1}^{N} c_n V_n + w_p, \tag{7}$$

where $V_n$ are Trefftz functions, $c_n$ are coefficients and $w_p$ is the particular solution of nonhomogeneous equation. To find $c_n$, the suitable functional which describes error of fulfilling the boundary conditions by approximate solutions in least square sense has to be minimized. The inhomogeneity in equation can be approximated by a linear combination of monomials and the particular solution is obtained using the inverse operator for monomials, where $L^{-1}$ is an inverse operator to $L$. It is given by the formulas:

$$L_1^{-1}(x^k y^l) = \frac{1}{(k+2)(k+1)}[x^{k+2}y^l - l(l-1)L^{-1}(x^{k+2}y^{l-2}), \tag{8}$$

$$L_2^{-1}(x^k y^l) = \frac{1}{(l+2)(l+1)}[x^k y^{l+2} - k(k-1)L^{-1}(x^{k-2}y^{l+2}). \tag{9}$$

The average of (8) and (9) is taken for calculations.

## 3. Example

In this section the presented method will be illustrated by an example. The quality of the method will be checked by comparing the approximation with the exact solution. Moreover, the errors will be compared with results presented in [5] where the same equation was solved using the Picard's iterations.

Let us consider a nonlinear problem of the stationary heat conduction described by the equation:

$$\frac{\partial}{\partial x}\left[e^u \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[e^u \frac{\partial u}{\partial y}\right] = 0 \quad \text{for} \quad (x,y) \in \Omega = (0,1) \times (0,1), \tag{10}$$

with boundary conditions:

$$u(0,y) = u_1(y) = ln(500\ cosy), \tag{11}$$

$$u(1,y) = u_2(y) = ln(500\ cosy) + 1, \tag{12}$$

$$u(x,0) = u_3(x) = ln(500\ e^{2x}) - x, \tag{13}$$

$$u(x,1) = u_4(x) = ln(500\ cos1\ e^{2x}) - x. \tag{14}$$

The exact solution for the problem described by (10) – (14) has the form:

$$u(x,y) = ln(500\ cosy\ e^{2x}) - x,$$

Equation (10) can be converted to the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 0. \tag{15}$$

In the equation *f(r)=0* and the operators *L* and *N* have the forms:

$$L = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

$$N = \left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2.$$

The linear part of operator *A* is the Laplace operator. The Trefftz functions are harmonic polynomials. They are obtained as the real and imaginary part of:

$$\left(\frac{x+iy}{n!}\right)^n.$$

To solve (15) we construct the following homotopy:

$$H(v,p) = (1-p)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial^2 u_0}{\partial y^2}\right] + p\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] = 0. \tag{16}$$

By choosing an initial approximation $u_0$:

$$u_0 = -\frac{1}{2}y^2 + x + ln\,500,$$

and substituting (5) into (16) and rearranging the resultant equation based on powers of p-terms, we get:

$$p^0: \quad \frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2} - \frac{\partial^2 u_0}{\partial x^2} - \frac{\partial^2 u_0}{\partial y^2} = 0,$$

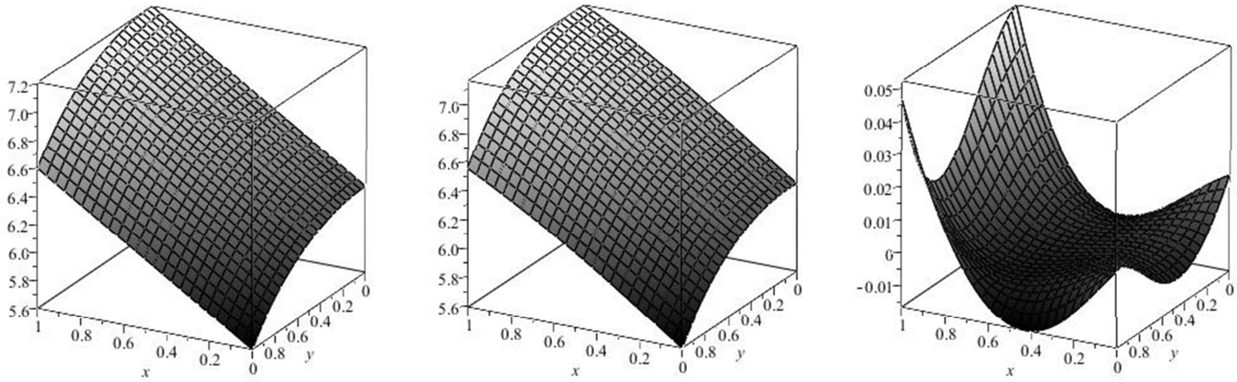
$$p^1: \quad \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2} + \left(\frac{\partial v_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial y}\right)^2 = 0,$$

$$p^2: \quad \frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} + 2\frac{\partial v_0}{\partial x}\frac{\partial v_1}{\partial x} + 2\frac{\partial v_0}{\partial y}\frac{\partial v_1}{\partial y} = 0,$$

$$p^3: \quad \frac{\partial^2 v_3}{\partial x^2} + \frac{\partial^2 v_3}{\partial y^2} + \left(\frac{\partial v_1}{\partial x}\right)^2 + 2\frac{\partial v_0}{\partial x}\frac{\partial v_2}{\partial x} + \left(\frac{\partial v_1}{\partial y}\right)^2 + 2\frac{\partial v_0}{\partial y}\frac{\partial v_2}{\partial y} = 0,$$

and so on. In each step the exact solution is approximated according to the formula (7).

Figure 1 shows the exact solution, approximation by 7 polynomials for $p^2$ and the error of approximation. It is clearly seen that the presented method gives very good results, the approximation is very close to the accurate solution and the error oscillates around 0.



(a) (b) (c)

**Fig. 1.** a) Exact solution, b) approximation for $p^2$ by 7 Trefftz functions, c) error of approximation.

Because the exact solution of the problem is known, the quality of the approximate solution can be checked by calculating the mean relative error:

$$E = \sqrt{\frac{\int_0^1 \int_0^1 (u(x,y) - v(x,y))^2 dxdy}{\int_0^1 \int_0^1 (u(x,y))^2 dxdy}} \cdot 100\% \, .$$

The values of the error $E$ depend on the number of Trefftz functions used in approximation and the number of homotopy perturbation steps. The results obtained using homotopy perturbation method are presented in Tab. 1. To compare the quality of this method the results obtained by Picard's iterations in [5] are shown in Tab. 2.

| Number of Trefftz functions | Number of steps | | |
|---|---|---|---|
| | $p^0$ | $p^1$ | $p^2$ |
| 5 | 0.35 | 1.40 | 0.26 |
| 7 | 0.35 | 1.33 | 0.15 |
| 9 | 0.31 | 1.32 | 0.26 |
| 11 | 0.30 | 1.17 | 0.25 |
| 13 | 0.30 | 1.03 | 0.22 |

**Tab. 1.** A mean relative error of approximation (*E[%])* – homotopy perturbation method.

| Number of Trefftz functions | Number of iterations | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| 5 | 1.09 | 0.25 | 0.19 | 0.18 |
| 7 | 1.08 | 0.13 | 0.06 | 0.03 |
| 9 | 0.91 | 0.16 | 0.10 | 0.11 |
| 13 | 0.92 | 0.10 | 0.04 | 0.04 |

**Tab. 2.** A mean relative error of approximation (*E[%])* – Picard's iteration.

In Tab. 1 we can observe that taking more Trefftz functions leads to better approximation. Although even for 5 polynomials the errors are very small. In general the error of approximation decreases after each step. For $p^2$ the results are very good and the errors do not exceed 0.3%, which proves a good quality of the homotopy perturbation method. In the first step the value of the error is relatively small because the approximation strongly depends on the initial solution which is very close to the exact solution.

Comparing homotopy perturbation method with Picard's iterations presented in the paper [5] we can see that both methods give very good results. Marginally greater errors were obtained by homotopy perturbation which is probably due to numerical errors.

## 4. Conclusion

Nonlinear engineering problems are very common and solving them is sometimes mathematically complex. In this paper homotopy perturbation method combined with the Trefftz functions was used to solve nonlinear stationary heat conduction problems. The main advantage of the presented method is in its mathematical simplicity. The homotopy equation can be easily constructed, and the solution procedure is simple. An approximate solution consists of a linear combination of Trefftz functions and a particular solution for a nonhomogeneous equation. A comparison of the results between the numerical and the exact solutions implies that we have achieved a good result. The error decreases with the increasing number of polynomials and iteration steps. A satisfactory approximation is obtained for $p^2$ even for a small number of Trefftz functions. The results obtained by homotopy perturbation method for the nonlinear stationary differential equation were compared with results obtained by Picard's iteration presented in [5]. The comparison shows a good agreement between received results. The results show that homotopy perturbation method is a powerful mathematical tool for solving nonlinear partial differential equations having wide applications in science and engineering.

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# "Well, I mean, it's sort of…" – Hesitancy and Court Interpreting

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**Abstract.** This paper deals with the topic of court interpreting and several procedural predicaments interpreters may come across in the courtroom. Specifically, interpreting hesitant, repetitive and incoherent answers of witnesses as well as prompting them to answer is focused on. Also, the hesitation on the part of the interpreters is pointed out. The international scholarly research is compared with the views of Czech and Slovak court interpreters with English as a working language. The survey has shown that the Czech and Slovak court interpreters indeed try to maintain the same style as the minority language speakers and do not prompt the witnesses to answer when they hesitate to do so.

**Keywords:** court interpreting, procedural predicaments, incoherence, hesitancy, repetition.

## 1. Introduction

Interpreting is defined in various ways by a number of scholars. First of all, it is essential to distinguish interpreting from translating, because these two processes are often perceived by laymen as the same one. Colin and Morris [1], claim that interpreting is a similar activity to translating, although different professional skills are needed for them. Both terms "refer to the process of turning a message in one (source) language into another (target) language. The term *translation* is used [...] where *written* material is re-expressed in another language in the written form; the term *interpretation* [...] is used where material presented orally is re-expressed *orally*."

In the same manner, Pöchhacker [2] defines interpreting as a *translational activity* which "can be distinguished from other types of translational activity most succinctly by its *immediacy*: In principle, interpreting is performed ˈhere and now' for the benefit of people who want to engage in communication across barriers of language and culture."

According to settings [3], interpreting might be divided into numerous subcategories, such as conference, media, business, medical, community, sign language and court interpreting. And it is the last one that will be focused on in the present paper. Because of the specific settings, there are also particular issues that might cause a difficulty for interpreters. Some of these will be pointed out later on.

## 2. Methodology

The findings discussed in the present paper are taken from a study which was conducted by means of an online questionnaire created in Google in April 2013. The survey targeted Czech and Slovak court interpreters, who were listed as having the language combinations Czech–English or Slovak–English. The website link to the questionnaire was distributed via e-mail to approximately 69 Slovak interpreters and 154 Czech interpreters, which makes a total of 223 court interpreters. There were 47 court interpreters, who responded to the questionnaire. However, not all responses could be used, because one respondent answered twice and one claimed to be bilingual, which may have falsified the data. In addition, one of the Slovak interpreters filled in the questionnaire, even though she had no experience with interpreting at court or for police, which could compromise the results as well. Therefore, only 45 responses were processed.

As far as the working languages of these selected court interpreters are concerned, apart from English, the most common working languages were German (4), Spanish (4), French (3) and Russian (2). Besides, there were some other languages as Italian, Thai, Portuguese, Finnish and Dutch.

These 45 court interpreters stated that besides the Czech Republic and Slovakia they had worked in the United Kingdom (17), the United States of America (4), Austria (2), Spain (2), Germany (1), Poland (1) and Thailand (1).

There was an interpreter with forty years of practice as well as one with only one year of experience in the field. The average length of their practice was thirteen years. Numerous interpreters stated that they had been working as interpreters for a longer time and had been appointed court interpreters only later.

## 3. Procedural Predicaments

During the process of interpreting in a court environment, there are many potential issues an interpreter has to face. In general, they can be divided into three categories – linguistic, procedural and ethical. Some predicaments encompassed in the second category are dealt with and explained in more detail below. Examples from scholars, who have done research in this particular area, are provided and subsequently, the data obtained from the questionnaire are presented.

### 3.1. Insertion of Hedges, Particles and Hesitation Forms

Hedge, as defined in dictionaries represents "a word or phrase, such as *possibly* or *I think*, that mitigates or weakens the certainty of a statement" [4]. Consequently, an interpreted testimony which contains inserted hedges does not sound as affirmative as the original [5] and creates a vague impression of the person who testifies. Also, it might create a more colloquial speech style "and give the answer a more logical connectedness to what the witness has just said in an immediately prior utterance" [5].

Berk-Seligson [5] depicts the archetypal hedge *sort of* in a case of an illegal alien who flew to Mexico. When asked about the appearance of the plane, he answers with certainty, which is unfortunately not reflected in the interpretation at all:
Witness: *Una avioneta pequeña blanca con rayitos azules.* (It was a small white airplane with blue stripes.)
Interpreter: It was a small airplane, white, with <u>a sort of, a sort of</u> blue lines, blue stripes.

Importantly, Berk-Seligson adds that hesitation forms like *uh* and *well* are frequently the interpreter's own product, inserted into their speech when they concentrate mentally on the process of interpreting and are under pressure. All these insertions are to be found in the example from Berk-Seligson [5]:
Attorney: Who else was with you?
Interpreter: *¿Quién más estaba con usted cuando usted cruzó?*
Witness: *Varios hondureños.* (Several Hondurans)
Interpreter: <u>Uh, uh</u>, several other Hondurans.
Attorney: Approximately how many?
Interpreter: *¿Aproximadamente cuántos en total?*
Witness: *Un promedio de veintiuno.* (An average of twenty-one.)
Interpreter: <u>Uh, probably</u> an average of twenty-one <u>people</u>.

Here, highlighting (an average of twenty-one *people*), i.e. a process "in which the interpreter adds material that is understood but not explicitly stated in the source language" [6], hedging (*probably*) and hesitation form (*uh*) work together, but not in favor of the witness. He was asked who else was present at the fence where he illegally entered the United States. Although his replies are confident in Spanish, the rendition does not produce the same impression and it lacks the certainty of the source language message. As a consequence, the witness and his testimony might be viewed as untrustworthy.

### 3.2. Witness's Hesitancy, Repetition and Incoherence

As has been already mentioned, various elements of speech style, such as hedges, are inserted by the interpreter into the rendition, many times only unconsciously. On the other hand, these very elements are often systematically omitted, as if "not salient enough for court interpreters to include them" [5].Hale [7] adds that discourse markers, words "…such as 'well,' 'now,' and 'see' that are common in everyday oral communication," get omitted as well. She adds that omissions or misinterpretations of these expressions can influence the illocutionary force of an utterance.

Witness: *No sé, o sea que, que que yo la veía y un poco asustado, yo no la puedo describir, así, como, como era, pero sí sé que era negra y, y, y como verde, así, ¿no?* (I don't know, I mean, that that that I saw it and I was a bit scared, I can't describe it, how, how, how it was, but I know it was black and, and, and like green, like this, you know?)

Interpreter: Eh, I… cannot describe it fully because at that moment I was frightened, it all happened so suddenly, I do remember the colour though, it was black, sort of greenish black.

In this excerpt the interpreter improves the wording of the witness's answer by making it more coherent and precise, which could not be viewed as a faithful rendition of the original message in Spanish.

In the questionnaire, the Czech and Slovak court interpreters were asked the following question: *The witness is obviously not very articulate and has difficulties expressing him- or herself. His or her speech is very repetitive, incoherent, hesitant and very colloquial. How do you interpret his or her answers?*

This question focuses on the accurate rendition of hesitancy markers in the witness's speech. As they are a sign of *powerless speech1* [5], it is important to keep them in the interpreted version of the utterance, so that the testimony style does not get improved or does not create a false image of the speaker. Moreover, the flow of the proceeding would get shortened, too, if they get omitted, and the level of the witness's trustworthiness will be changed. 66 % of Hale's respondents would attempt to maintain the style of the witness's answers. 51 % of the Czech and Slovak court interpreters would try to maintain the same testimony style, too. 27 % of the respondents would concentrate more on the meaning and re-express it in the best way they can, 18 % would on one hand improve the style, but on the other hand inform the judge about the difficulties the witness has expressing him– or herself. For better insight it is worthy to include the opinion of Jana Nováčková, one of the interpreters who chose the "other" option: "If it is clear that it is not me who has the problem to convey the meaning, I maintain the same style as the speaker. If the court is in a hurry – which can happen – I ask the speaker if I understand it right and summarize what he agreed on or let him correct what he feels that should be corrected. However, before doing that I ask the judge for permission to "concentrate on the meaning of the answer". The judges usually have some knowledge of English and are able to make decision in what way they would like to have it done. Generally, it cannot be recommended that interpreters provide summary of relevant and coherent facts. This is usually done by the judge who dictates the parts of the witness's speech relevant for the legal evaluation of the issue to the recording clerk."

1 Term coined by O'Barr et al [8]. It is a speech style that comprises a set of speech traits that make the speaker sound less competent, truthful, convinging, intellingent and trustworthy. This style contains a lot of hedges, superpolite forms, tag questions, hypercorrect grammar, special lexicon and other traits.

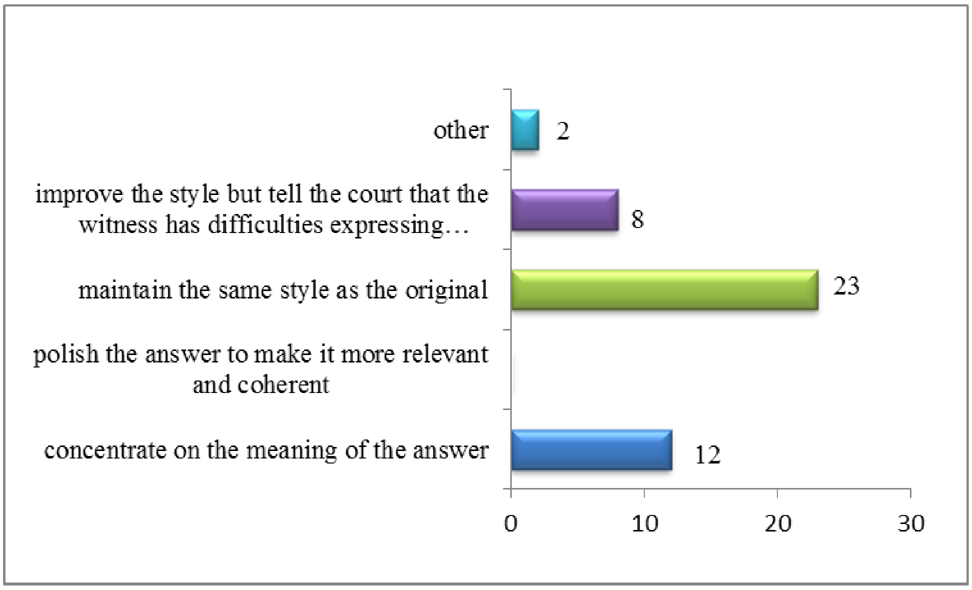


**Fig. 1.** Interpreting Hesitant and Incoherent Answers

### 3.3. Prompting Witnesses or Defendants to Speak

Another issue regarding hesitancy in the courtroom is an occasional urge to prompt the witness or defendant to answer, when they are hesitant do to so. "Interpreters use certain prompting mechanisms to speed up a witness's reply to an attorney's question, or to accelerate a response to the routine "Do you understand?" type of question used by judges to address defendants during arraignments, changes of plea, and sentencings" [5]. That was also the case with an example from Berk-Seligson's [5] study, when the interpreter prompted the witness to answer a question, which the witness replied with affirmative *Mhm*. only. Since that cannot be noted in the official record from the proceeding, the witness was urged to "answer *yes* or *no*" by the interpreter. However, only the judge is the one who shall request that kind of an answer.

Prosecuting attorney: And you had no papers or documents allowing you to come in lawfully, is that correct?
Interpreter: *¿Y Ud. no tenía documentos o papeles que lo autorizaran a entrar legalmente a los Estados Unidos? ¿Es esto correcto?*
Witness: *Mhm.*
Interpreter [addressing witness]: *¡Conteste!* (Answer!)
Witness: *¿Que no tenía papeles yo de migración de este lado?*
Interpreter: You mean I didn't have any immigration papers from this side?
Prosecuting Attorney: Right.
Interpreter: *Sí.*

In addition, prompting witnesses to answer occurs more often, when multiple witnesses or defendants are present. "Perhaps because often no one defendant is singled out, but instead, all are asked the same question at the same moment, defendants become reticent" [5].That kind of situation is depicted in the following excerpt by Berk-Seligson, where the speakers are testifying against a smuggler:

Judge: As witnesses each of you is entitled to be represented by an attorney. Do any of you have an attorney?
Interpreter: *Come testigos que son, tienen Uds. el derecho de ser representados por un abogado. ¿Alguno de Uds. tiene abogado que los represente?*
Interpreter [addressing defendants]: *Conteste por favor.* (Please answer.)

Defendants: *No*.
Interpreter [addressing judge]: All answered "No" sir.

Importantly, Berk-Seligson [5] raises the question about the effect of such behavior of interpreters, which is a factor not to be underestimated. "In reality, if a witness or defendant does not answer a question quickly, such hesitancy can often be a symptom of failure to comprehend the question. A frightened witness or defendant may not feel confident enough to say, "I don't understand."' She goes on and speculates, whether the pressure put on them pushes them into answering prematurely, even without being sure about the meaning of the question.

In the survey, Czech and Slovak court interpreters were asked: *Do you prompt the witness or the defendant to answer questions, when they are hesitant about the reply?* 96 % of the respondents replied that they do not use such mechanism in the courtroom and thus they would not control the flow of the testimony by accelerating the responses of the witness or the defendants. In both the Czech and Slovak group there is only one respondent in each one, who does prompt the testifying person to answer.

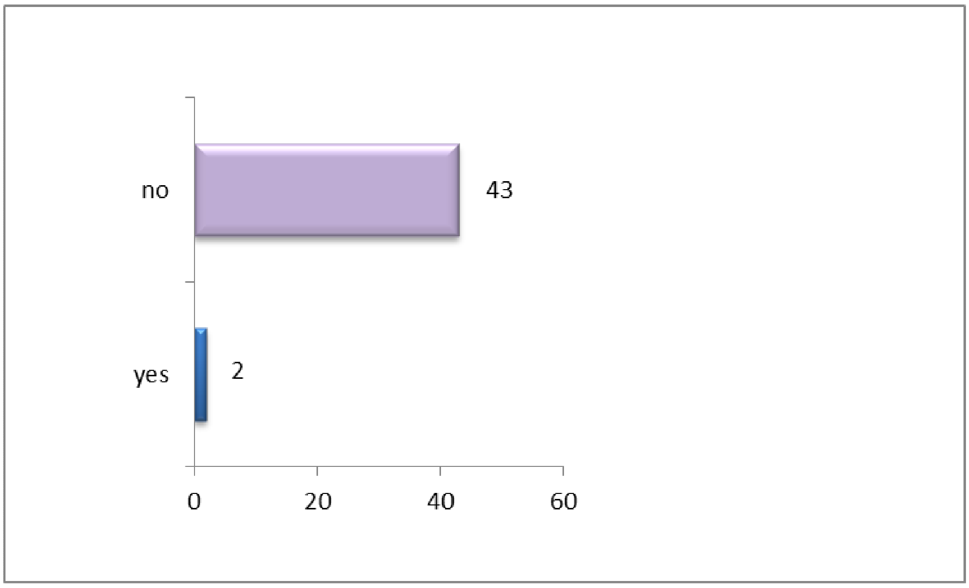


**Fig. 2.** Prompting the Witnesses or Defendants to Speak

## 4. Conclusion

It can be stated that hesitancy is to be found in the courtroom in various ways – either on the part of the witnesses or defendants or unconsciously, on the part of the interpreters.

Furthermore, it might be concluded that in practice, Czech and Slovak court interpreters' experience corresponds with the scholarly research of Berk-Seligson and other researchers in court interpreting as a linguistic area. Not only do the Czech and Slovak interpreters try to maintain the hedges and hesitancy in their renditions, but they also do not prompt the witnesses or defendants to answer if they are hesitant to do so. As a result, testimonials can be rendered faithfully, maintaining the same style as the witnesses adopt, with the hesitancy discourse markers kept in the speech.

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# Bounded Solutions of k-Dimensional System of Difference Equations

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**Abstract.** The k - dimensional system of neutral type nonlinear difference equations with delays in the following form

$$\begin{cases} \Delta\big(x_i(n)+p_i(n)x_i(n-\tau_i)\big)=a_i(n)\,f_i(x_{i+1}(n-\sigma_i))+g_i(n), & i=1,\dots,k-1 \\ \Delta\big(x_k(n)+p_k(n)x_k(n-\tau_k)\big)=a_k(n)\,f_k(x_1(n-\sigma_k))+g_k(n) \end{cases}$$

is considered. The aim of this paper is to present sufficient conditions for the existence of nonoscillatory bounded solutions of the system above with various sequences $p_1,\dots,p_k$.

**Keywords:** System of difference equation, delays, k -dimensional, neutral type, nonoscillatory solutions, boundedness.

## 1. Introduction

In this paper we consider a nonlinear difference system of the *k* equations of the form

$$\begin{cases} \Delta\big(x_i(n)+p_i(n)x_i(n-\tau_i)\big)=a_i(n)\,f_i(x_{i+1}(n-\sigma_i))+g_i(n), & i=1,\dots,k-1 \\ \Delta\big(x_k(n)+p_k(n)x_k(n-\tau_k)\big)=a_k(n)\,f_k(x_1(n-\sigma_k))+g_k(n) \end{cases} \tag{1}$$

where $\Delta$ is the forward difference operator defined by $\Delta u(n)=u(n+1)-u(n)$.

Here $R$ is a set of real numbers, $N=\{0,1,2,\dots\}$ and $[\sigma_1,\dots,\sigma_k],[\tau_1,\dots,\tau_k]\in N^k$. By $n_0$ we denote $max\{\tau_1,\dots,\tau_k,\sigma_1,\dots,\sigma_k\}$ and $N_0=\{n_0,n_0+1,\dots\}$.

Moreover $a_1,\dots,a_k$, $g_1,\dots,g_k$, $p_1,\dots,p_k$ are the sequences of real numbers and a function $[f_1,\dots,f_k]:\mathrm{R}^k\to R^k$. $x_1,\dots,x_k$ are real sequences defined for $n\in N$. Throughout this paper $X(n)$ denotes a vector $[x_1(n),\dots,x_k(n)]$. For the elements of $R^k$ the symbol $|\cdot|$ stands for the maximum norm. By $B$ we denote the Banach space of all bounded sequences in $R^k$ with the supremum norm, i.e.

$$B=\left\{X:\mathrm{N}\to R^k : \|X\|=\sup_{n\in N}|X(n)|<\infty\right\}.$$

A sequence of real numbers is said to be nonoscillatory if it is either eventually positive or eventually negative. By a solution of system (1) we mean a sequence $(X(n))$ which satisfy system (1) for sufficiently large *n.* The solution $X$ of the system (1) is called nonoscillatory if all its components, i.e. $x_1,\dots,x_k$ are nonoscillatory.

We establish sufficient conditions for the existence of nonoscillatory bounded solutions of the considered system with various sequences $p_1,\dots,p_k$. The results are illustrated by examples.

The following definition and theorem will be used in the sequel

**Definition 1** ( Uniformly Cauchy subset, [3] ). *A subset* $\Omega$ *of the Banach space* $B$ *is said to be uniformly Cauchy if for every* $\varepsilon > 0$ *there exists a positive integer* $N$ *such that* $|X(i) - X(j)| < \varepsilon$ *whenever* $i, j > N$ *for any* $X \in \Omega$.

**Theorem 1** (Krasnoselskii's Fixed Point Theorem, [4]). *Let* $B$ *be a Banach space, let* $\Omega$ *be a bounded closed convex subset of* $B$ *and let* $F, T$ *be maps of* $\Omega$ *into* $B$ *such that* $Fx + Ty \in \Omega$ *for every pair* $x, y \in \Omega$. *If* $F$ *is a contraction and* $T$ *is completely continuous, then the equation* $Fx + Tx = x$ *has a solution in* $\Omega$.

## 2. Existence of Bounded Nonoscillatory Solution

In this section, using the Krasnoselskii's fixed point theorem we establish sufficient conditions for the existence of nonoscillatory bounded solutions of the system (1).

**Theorem 2.** *Assume that for* $i = 1, \dots, k$

$$\sum_{n=1}^{\infty} |a_i(n)| < \infty, \tag{2}$$

$$\sum_{n=1}^{\infty} |g_i(n)| < \infty, \tag{3}$$

$$f_i : \mathrm{R} \to R \text{ is continuous function.} \tag{4}$$

*If there exist positive real numbers* $c_{p_i}$, $i = 1, \dots, k$ *such that*

$$0 < p_i(n) \le c_{p_i} < 1, \qquad n \in N_0, \tag{5}$$

*or*

$$-1 < -c_{p_i} \le p_i(n) < 0, \qquad n \in N_0, \tag{6}$$

*then the system (1) has a bounded nonoscillatory solution.*

*Proof:* For the fixed positive real number $r$ we define the set

$$\Omega = \left\{ X \in B : \frac{1}{8}(1 - c_{p_i})r \le |x_i(n)| \le r,\ i = 1, \dots, k,\ n \in N \right\}.$$

$\Omega$ is bounded closed convex subset of the Banach space $B$.

Since condition (4) is satisfied, we can take

$$M_f = \max_{i=1,\dots,k} \left\{ |f_i(t)| : t \in \left[ \frac{1}{8}(1 - c_{p_i})r, r \right] \right\}.$$

Assume that $M_f > 0$. From (2) and (3), there exists such $n_1 \in N_0$ that

$$\sum_{n=n_1}^{\infty} |a_i(n)| \le \frac{(1 - c_{p_i})r}{8M_f}, \qquad \sum_{n=n_1}^{\infty} |g_i(n)| \le \frac{(1 - c_{p_i})r}{4}.$$

Let $I_1, I_2, I_3, I_4$ be the subsets of the set $\{1, \dots, k\}$ and moreover, $I_i \cap I_j = \varnothing$ for $i \ne j$, $i, j = 1, 2, 3, 4$ and $I_1 \cup I_2 \cup I_3 \cup I_4 = \{1, \dots, k\}$.

The set $\Omega$ contains both nonoscillatory and oscillatory sequences, but we are only interested in the existence of bounded nonoscillatory solutions of (1). For this reason we consider four cases in the proof:

1. $\begin{cases} 0 < p_i(n) \le c_{p_i} < 1, & \text{for} \quad i \in I_1, \quad n \in N_0 \\ x_i(n) > 0, & \text{for} \quad i \in I_1, \quad n \in N_0 \end{cases}$

2. $\begin{cases} -1 < -c_{p_i} \le p_i(n) < 0, & \text{for} \quad i \in I_2, \quad n \in N_0 \\ x_i(n) < 0, & \text{for} \quad i \in I_2, \quad n \in N_0 \end{cases}$

3. $\begin{cases} 0 < p_i(n) \le c_{p_i} < 1, & \text{for} \quad i \in I_3, \quad n \in N_0 \\ x_i(n) < 0, & \text{for} \quad i \in I_3, \quad n \in N_0 \end{cases}$

4. $\begin{cases} -1 < -c_{p_i} \le p_i(n) < 0, & \text{for} \quad i \in I_4, \quad n \in N_0 \\ x_i(n) > 0, & \text{for} \quad i \in I_4, \quad n \in N_0 \end{cases}.$

Next, we define the maps $F, T : \Omega \to B$ where

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_k \end{bmatrix}, \qquad T = \begin{bmatrix} T_1 \\ \vdots \\ T_k \end{bmatrix},$$

$$(F_i X)(n) = (F_i X)(n_1) \quad \text{for} \quad i = 1, \ldots, k, \quad 0 \le n < n_1,$$
$$(F_i X)(n) = -p_i(n) x_i(n - \tau_i) + \frac{(1 + c_{p_i})r}{2} \quad \text{for} \quad i \in I_1 \cup I_2, \quad n \ge n_1,$$
$$(F_i X)(n) = -p_i(n) x_i(n - \tau_i) + \frac{(1 - c_{p_i})r}{2} \quad \text{for} \quad i \in I_3 \cup I_4, \quad n \ge n_1,$$
$$(T_i X)(n) = (T_i X)(n_1) \quad \text{for} \quad i = 1, \ldots, k, \quad 0 \le n < n_1,$$
$$(T_i X)(n) = -\sum_{s=n}^{\infty} a_i(s)\ f_i(x_{i+1}(s - \sigma_i)) - \sum_{s=n}^{\infty} g_i(s) \quad \text{for} \quad i = 1, \ldots, k-1, \quad n \ge n_1, \tag{7}$$
$$(T_k X)(n) = -\sum_{s=n}^{\infty} a_k(s)\ f_k(x_1(s - \sigma_k)) - \sum_{s=n}^{\infty} g_k(s) \quad \text{for} \quad n \ge n_1.$$

We have to show that $F$ and $T$ satisfy the assumptions of Theorem 1. First we can prove that if $X, \overline{X} \in \Omega$ , then $FX + T\overline{X} \in \Omega$.

Then it is easy to see that $F$ is a contraction mapping

$$\| FX - F\overline{X} \| \le \max_{i=1,\ldots,k} \{c_{p_i}\} \cdot \| X - \overline{X} \|,$$

where $X, \overline{X} \in \Omega$, $n \ge n_1$, and by (5) and (6) there is $0 < \max_{i=1,\ldots,k} \{c_{p_i}\} < 1$.

The next step is to show continuity of $T$. Let $X_j = [x_{1j}, \ldots, x_{kj}] \in \Omega$ for $j \in N$ and for $i = 1, \ldots, k$ there is $x_{ij}(n) \to x_i(n)$ as $j \to \infty$.

Since $\Omega$ is closed, we have $X = [x_1, \ldots, x_k] \in \Omega$. By (7), (2) and (4) we obtain for $i = 1, \ldots, k-1$

$$\left| (T_i X_j)(n) - (T_i X)(n) \right| \le \sum_{s=n}^{\infty} |a_i(s)|\, |\, f_i(x_{i+1\,j}(s - \sigma_i)) - f_i(x_{i+1}(s - \sigma_i))\, | \to 0 \_\text{if} \quad j \to \infty,$$

where $n \in N$ . There is analogous inequality for $T_k$. Therefore $\| (TX_j) - (TX) \| \to 0 \_\text{if}\ j \to \infty$.

Next we will demonstrate that $T\Omega$ is uniformly Cauchy. By [3] it means that $T\Omega$ is relatively compact. Let $X \in \Omega$ . We conclude from the assumptions (2), (3) and (4) that for any given $\varepsilon > 0$ there exists such an integer $n_2 > n_1$ that for $n \ge n_2$ we have

$$\sum_{s=n}^{\infty}|a_i(s)||f_i(x_{i+1}(s-\sigma_i))|+\sum_{s=n}^{\infty}|g_i(s)|<\frac{\varepsilon}{2},\quad i=1,\ldots,k-1.$$

Hence for $n_4 > n_3 \geq n_2$ we obtain

$$|(T_iX)(n_4)-(T_iX)(n_3)|=\left|\sum_{s=n_4}^{\infty}a_i(s)\ f_i(x_{i+1}(s-\sigma_i))+\sum_{s=n_4}^{\infty}g_i(s)-\sum_{s=n_3}^{\infty}a_i(s)\ f_i(x_{i+1}(s-\sigma_i))-\sum_{s=n_3}^{\infty}g_i(s)\right|<\varepsilon.$$

By Theorem 1, there exists a sequence $(X(n))$ such that $(FX)(n)+(TX)(n)=X(n)$ . Finally, we can easily verify that $(X(n))$ satisfies the system (1) for $n \geq n_1$.

The proof is complete.

**Example 1.** *Consider the difference system*

$$\Delta\left(x_1(n)+\frac{1}{2n}x_1(n-1)\right)=\frac{5n^4-21n^3+22n^2+4n-8}{4n^6-16n^5+10n^4+16n^3-14n^2}\,x_2(n-2)+\frac{1}{n^2}$$

$$\Delta\left(x_2(n)-\frac{1}{2n}x_2(n-2)\right)=\frac{2n^5-17n^4+43n^3-48n^2+27n-7}{2n^8-4n^7-6n^6+8n^5+8n^4}\,x_3^3(n-1)-\frac{1}{n^2}$$

$$\Delta\left(x_3(n)+\frac{1}{2n}x_3(n-1)\right)=\frac{3n^3-3n^2+2}{4n^5-2n^4-6n^3}\,x_4(n-1)+\frac{1}{n^3}$$

$$\Delta\left(x_4(n)-\frac{1}{2n}x_4(n-1)\right)=\frac{2n^2-5n+3}{n^4+n^3}\,x_1^2(n-1).$$

*All assumptions of Theorem 2 are satisfied. One of the bounded solutions of the above system is* $X(n)=\left[1+\frac{1}{n},-2+\frac{1}{n^2},-1-\frac{1}{n},2-\frac{1}{n}\right].$

We will work at the more general case than in Theorem 2, that assume also another values of $[(p_1(n)),\ldots,(p_k(n))]$. Let $I_5,I_6,I_7,I_8$ be the subsets of the set $\{1,\ldots,k\}$ that $I_i\cap I_j=\varnothing$ for $i\neq j$, $i,j=5,6,7,8$ and $I_5\cup I_6\cup I_7\cup I_8=\{1,\ldots,k\}$.

**Theorem 3.** *Let assumptions (2), (3) and (4) hold. If there exist positive real numbers* $c_{p_i}$, $i\in I_5\cup I_6$ *and* $\tilde{c}_{p_i}$ , $i\in I_7\cup I_8$ *that satisfy inequalities:*

$$0<p_i(n)\leq c_{p_i}<1,\qquad \text{for}\quad i\in I_5,\quad n\in N_0,$$

$$-1<-c_{p_i}\leq p_i(n)<0,\qquad \text{for}\quad i\in I_6,\quad n\in N_0,$$

$$1<\tilde{c}_{p_i}\leq p_i(n),\qquad \text{for}\quad i\in I_7,\quad n\in N_0,$$

$$p_i(n)\leq-\tilde{c}_{p_i}<-1,\qquad \text{for}\quad i\in I_8,\quad n\in N_0,$$

*then the system (1) has a bounded nonoscillatory solution.*

**Example 2.** *Consider the difference system of eight equations*

$$\Delta\left(x_1(n)+\frac{1}{3n}x_1(n-1)\right)=\frac{-8n^4+35n^3-41n^2+12}{12n^6-60n^5+63n^4+60n^3-75n^2}\,x_2^2(n-2)+\frac{1}{n^2}$$

$$\Delta\left(x_2(n)-\frac{1}{2n}x_2(n-2)\right)=\frac{-4n^3+26n^2-51n+27}{4n^5-18n^4+16n^3+18n^2-20n}\,x_3(n-3)$$

$$\Delta\left(x_3(n)+\frac{1}{3n}x_3(n-2)\right)=\frac{-5n^4+10n^3-3n^2+3n-6}{9n^6-15n^5-12n^4+12n^3}\,x_4(n-1)-\frac{1}{n^3}$$

$$\Delta\left(x_4(n)-\frac{1}{2n}x_4(n-3)\right)=\frac{-15n^4+89n^3-136n^2-48n+160}{8n^6-40n^5+42n^4+36n^3-54n^2}\,x_5^2(n-2)+\frac{7}{n^2}$$

$$\Delta\left(x_5(n)+\left(4+\frac{1}{n}\right)x_5(n-1)\right)=\frac{n^4+4n^3-27n^2+28n+4}{4n^6-12n^5+5n^4+12n^3-9n^2}\,x_6^2(n-2)+\frac{1}{n^2}$$

$$\Delta\left(x_6(n)+\left(-4+\frac{1}{n}\right)x_6(n-3)\right)=\frac{22n-22}{2n^4-7n^3+9n}\,x_7(n-2)$$

$$\Delta\left(x_7(n)+\left(4+\frac{1}{n}\right)x_7(n-2)\right)=\frac{4n^5-48n^4+179n^3-198n^2-81n+108}{4n^7-28n^6+61n^5-22n^4-65n^3+50n^2}\,x_8^2(n-3)-\frac{6}{n^2}$$

$$\Delta\left(x_8(n)+\left(-4-\frac{1}{n}\right)x_8(n-2)\right)=\frac{-13n^4+40n^3-24n^2-22n+28}{4n^6-12n^5+5n^4+12n^3-9n^2}\,x_1^2(n-2)+\frac{7}{n^2}.$$

*One of the bonded nonoscillatory solutions is*

$$X(n)=\left[2+\frac{1}{n},2-\frac{1}{n},-2-\frac{1}{n},-3-\frac{1}{n},3-\frac{1}{n},3+\frac{1}{n},-3+\frac{1}{n},-3-\frac{1}{n}\right].$$

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# Non-Oscillatory Solutions of a Neutral Difference Equations

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**Abstract.** There will be presented a some classification of non-oscillatory solutions of the considered system and sufficient conditions for solutions to be bounded or unbounded. The obtained results will be illustrated by examples.

**Keywords:** difference equation, neutral type, nonlinear system, non-oscillatory, bounded, unbounded solution.

## 1. Introduction

In the paper, we consider a nonlinear $k$-dimensional neutral difference system of equations of the form

$$\Delta\big(X(n) + P(n)X(n-\delta)\big) = A(n)F\big(X(n-\tau)\big), \quad n \geq n_0 \qquad (1)$$

where $n_0 \in N$, $\delta$ and $\tau$ are nonnegative integers. Here

$$X(n) = (x_1(n), x_2(n), \dots, x_k(n))^T : Z^{\infty}_{n_0-\eta} \to R_k,$$
$$X_i = (X_i(n)), \qquad i = 1,2, \dots, k,$$

and

$$X = (x_1, x_2, \dots, x_k)$$

is an unknown vector, $Z^{\infty}_{n_0-\eta}$:={$n_0 - \eta$,$n_0 - \eta + 1$,...}, $\eta = \max\ \{\delta, \tau\}$, $\Delta$ is the forward difference operator, it means $\Delta X(n) = X(n+1) - X(n)$, $P$, $A: Z^{\infty}_{n_0} \to R^k \times R^k$ are square matrices such that

$$P(n) := diag(p(n), 0, \dots, 0), \qquad A(n) := diag(a_1(n), a_2(n), \dots, a_k(n)),$$

and a function $F : R^k \to R^k$ is specified in the following way

$$F(X(n)) := (f_1(x_2(n)), f_2(x_3(n)), \dots, f_{k-1}(x_k(n)), f_k(x_1(n)))^T$$

where $f_i: R \to R$ are functions such that

$$uf_i(u) > 0 \ \ for \ \ u \neq 0, \qquad i = 1, \dots, k. \qquad (2)$$

We assume

$$\lim_{n\to\infty} p(n) = p \in R \ \ and \ \ |p| < 1, \qquad (3)$$

and

$$a_i: Z_{n_0}^{\infty} \to R_+ \cup \{0\}, \quad R_+ := (0,\infty), \quad \sum_{n=1}^{\infty} a_i(n) = \infty, \quad i = 1,\ldots,k-1, \quad (4)$$

$$a_k: Z_{n_0}^{\infty} \to R_+. \quad (5)$$

The presented here results are generalisation to $k$-dimensional system our previous results obtained for three and four dimensional systems in [1] and [2] respectively.

Let us begin with recalling the basic definitions.

**Definition 1**
A vector $X,\ n \in Z_{n_0-\eta}^{\infty}$ is a solution of (1) if it satisfies (1) for every $n \in Z_{n_0}^{\infty}$.

**Definition 2**
A solution $X,\ n \in Z_{n_0-\eta}^{\infty}$ of (1) is eventually non-oscillatory if all its components are non-oscillatory for all sufficiently large $n$. Otherwise, it is called oscillatory.

**Definition 3**
A solution $X,\ n \in Z_{n_0-\eta}^{\infty}$ of (1) is said to be bounded if all its components are bounded. Otherwise, it is called unbounded.

Our goal is to study of boundedness and unboundedness of eventually non-oscillatory solutions of system (1).

## 2. Main Results

Before we state sufficient conditions for the solutions to be bounded or unbounded, we give all possibilities for sign of components of every eventually non-oscillatory solution $X$, $n > n_0 - \eta$ of system (1). We noticed that a classification of non-oscillatory solutions depends on whether the number of equations in system is odd or even.

**Theorem 1**
Let conditions (2) – (5) be satisfied.

(I) If a dimension $k$ of system (1) is even number then eventually non-oscillatory solutions $X,\ n \in Z_{n_0-\eta}^{\infty}$ satisfy exactly one of the following cases

- $\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) = \ldots = \operatorname{sgn} x_k(n)$
- $\operatorname{sgn} x_1(n) \neq \operatorname{sgn} x_2(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$
- $\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) = \operatorname{sgn} x_3(n) \neq \operatorname{sgn} x_4(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$

$\vdots$

- $\operatorname{sgn} x_1(n) = \ldots = \operatorname{sgn} x_{2i-1}(n) \neq \operatorname{sgn} x_{2i}(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$

$\vdots$

- $\operatorname{sgn} x_1(n) = \ldots = \operatorname{sgn} x_{k-1}(n) \neq \operatorname{sgn} x_k(n)$.

(II) If a dimension $k$ of system (1) is odd number then eventually non-oscillatory solutions $X,\ n \in Z_{n_0-\eta}^{\infty}$ satisfy exactly one of the following cases

- $\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) = \ldots = \operatorname{sgn} x_k(n)$
- $\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) \neq \operatorname{sgn} x_3(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$
- $\operatorname{sgn} x_1(n) = \ldots = \operatorname{sgn} x_4(n) \neq \operatorname{sgn} x_5(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$

$\vdots$

- $\operatorname{sgn} x_1(n) = \ldots = \operatorname{sgn} x_{2i}(n) \neq \operatorname{sgn} x_{2i+1}(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$

$\vdots$

- $\operatorname{sgn} x_1(n) = \ldots = \operatorname{sgn} x_{k-1}(n) \neq \operatorname{sgn} x_k(n)$.

Proof of this theorem is reduced to exclude all cases where components of eventually non-oscillatory solutions $X(n)$ satisfy the following conditions

$$x_i(n) > 0, \qquad x_{i+1}(n) < 0, \qquad x_{i+2}(n) < 0, \qquad \text{for some } i \in \{1, \ldots, k-2\}$$

or

$$x_i(n) < 0, \qquad x_{i+1}(n) > 0, \qquad x_{i+2}(n) > 0, \qquad \text{for some } i \in \{2, \ldots, k-2\}.$$

We present simple example of system (1) which show that the sets of solutions mentioned in Theorem 1 are nonempty.

**Example 1**

Let $n_0 > 2$ and $4 \leq l \leq k$ such that $l + k$ is even number. We put in $k$-dimensional system (1)

$$a_i(n) = \begin{cases} \frac{9}{8}, & i = 1 \\ 1, & i \in \{2, \ldots, l-3\} \\ 4^n, & i = l-2 \\ \frac{1}{2}, & i \in \{l-1, \ldots, k-1\} \\ \frac{1}{2^{2n+1}}, & i = k, \end{cases}$$

$$f_i(t) = t \quad \text{for} \quad 1 \leq i \leq k,$$

$$p(n) = \frac{1}{2}, \qquad \tau = 0, \qquad \delta = 2.$$

It is easy to see that a vector $X(n)$ with

$$x_1(n) = x_2(n) \ldots = x_{l-2}(n) = 2^n,$$

$$x_{l-1}(n) = x_{l+1}(n) = x_{l+3}(n) = \ldots = x_{k-1}(n) = \frac{1}{2^n},$$

$$x_l(n) = x_{l+2}(n) = \ldots = x_k(n) = -\frac{1}{2^n}$$

is one of unbounded solutions of system (1).

Now, we formulate theorems concerning on bounded and unbounded solutions of system (1). The proofs of them will be omitted, but it is worth to mention that lemmas obtained in [3] and in [4] were exploited for proving the main results of this paper.

**Theorem 2**
If conditions (2) – (5) are satisfied then every eventually non-oscillatory solution $X,\ n\ \in Z^{\infty}_{n_0-\eta}$ of (1) fulfilling condition

$$\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) = \operatorname{sgn} x_3(n)$$

is unbounded.

**Theorem 3**
If conditions (2) – (5) are satisfied then every eventually non-oscillatory solution $X,\ n\ \in Z^{\infty}_{n_0-\eta}$ of (1) fulfilling condition

$$\operatorname{sgn} x_1(n) \neq \operatorname{sgn} x_2(n)$$

is bounded.

Comparing all possibilities of changing the sign of components of eventually non-oscillatory solutions (mentioned in Theorem 1) with cases included in Theorems 2 and 3, it is easy to see that we have to consider only the case

$$\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) \neq \operatorname{sgn} x_3(n) \neq \operatorname{sgn} x_4(n) \neq \ldots \neq \operatorname{sgn} x_k(n)$$

which holds for systems with odd number of equations.

**Theorem 4**
If conditions (2) – (5) are satisfied then every eventually non-oscillatory solution $X,\ n\ \in Z^{\infty}_{n_0-\eta}$ of (1) fulfilling condition

$$\operatorname{sgn} x_1(n) = \operatorname{sgn} x_2(n) \neq \operatorname{sgn} x_3(n)$$

can be bounded or unbounded.

Thesis of the above theorem is confirmed by the following examples.

**Example 2**
Let $n_0 > 1$ and $k \geq 3$ is odd number. We take in $k$-dimensional system (1)

$$p(n) = \frac{n-1}{2n}, \qquad \tau = 0, \qquad \delta = 1, \qquad a_1(n) = \frac{7}{2n+2}, \qquad a_k(n) = \frac{1}{(n+1)(4n-1)},$$

$$a_i(n) = \frac{1}{n+1}, \qquad \text{for} \quad 2 \leq i \leq k-1,$$

$$f_i(t) = t, \qquad \text{for} \quad 1 \leq i \leq k.$$

It is easy to see that a vector $X$ with

$$x_1(n) = 4 - \frac{1}{n}, \qquad x_i(n) = \frac{(-1)^i}{n}, \qquad \text{for} \ \ 2 \leq i \leq k,$$

is one of the bounded solutions of system (1).

**Example 3**
Let $n_0 > 2$ and $k \geq 3$ is odd number. We take in $k$-dimensional system (1)

$$p(n) = \frac{1}{2}, \qquad \tau = 1, \qquad \delta = 2, \qquad a_1(n) = \frac{9 \cdot 4^n}{16}, \qquad a_k(n) = \frac{1}{4^n}$$

$$a_i(n) = \frac{1}{4}, \qquad \text{for} \quad 2 \le i \le k-1,$$

$$f_i(t) = t, \qquad \text{for } 1 \le i \le k.$$

It is easy to see that a vector $X$ with

$$x_1(n) = 2^n, \quad x_i(n) = \frac{(-1)^i}{2^n} \quad , \qquad \text{for} \quad 2 \le i \le k,$$

is one of the unbounded solutions of system (1).

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