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SECTION 8

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Effect of Fenton's Reagent on the Surface Functional Groups of Activated Carbon WD-extra Used in the Naphthol Green B Adsorption Process

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Abstract. The paper presents the impact of the regeneration by Fenton's reagent on commercial activated carbon WD-extra at different times on its surface functional groups. The adsorbent was used in the acid dye adsorption - naphthol green B. In the study, the concentration of acidic and basic functional groups on the surface of the activated carbon by Boehm method was determined. According to the experimental results it can be clearly concluded that regeneration with the Fenton's reagent increases the content of acid groups and a reduces the basic groups. Amount of acid groups has increased to about 2.78 mmol / g, and the base has decreased to 0.375 mmol/g for carbon at 8 hours of regeneration.

Keywords: adsorption, functional groups, activated carbon, Fenton's reagent

1. Introduction

Dyes are organic compounds used for coloring various products in the pharmaceutical, cosmetic, paper, textile, and other [1]. Some of these compounds are toxic, carcinogenic or mutagenic, and stop the penetration of light into the aqueous solution, thereby blocking photosynthesis. These facts make, that they must be removed from the wastewater before it reaches the receiving body of water such as lakes or rivers [2-3].

One of the most popular and most effective methods for removal of dyes from aqueous solutions is adsorption on activated carbons. This method is characterized by high efficiency, simplicity of design technology, high efficiency with respect to various groups of adsorbates [2,4]. The functional groups on the surface of activated carbons determine their adsorption properties such as electrochemical, catalytic, acid-base, oxidation-reduction, the hydrophilic-hydrophobic, and others. Surface oxygen functional groups can be divided into two types [5]:

• acidic - neutralized by the rule: carboxyl, hydroxyl, phenolic, quinone type carbonyl, lactone normal type fluorescein lactone, anhydride coming from a neighboring carboxyl groups,
• a basic - neutralized by acids chromenes, pyrones.

Spent activated carbons in the dye adsorption process become hazardous waste which must be subjected to a disposal or recycling [6]. Carbon sorbents can be subjected to repeated regeneration without losing their adsorption capacities [7]. One method of utilizing chemical regeneration hydroxyl radicals is Fenton reaction [8].

The aim of this study was to verify the impact of regeneration (at different times) of commercial activated carbon WD-extra Fenton reagent on the surface functional groups. Selected carbon sorbent was used in the process of adsorption of an acidic dye that is naphthol green B.

2. Methods of Investigation

2.1. Materials

Dye: Acid dye – naphthol green B – was used in the study, with the molecular formula $C_{30}H_{15}FeN_3Na_3O_{15}S_3$ and molar mass 878.79 g/mol. It was used to dye wool, nylon and paper and present it in the printing and dyeing wastewater [9].

Adsorbent: An activated carbon WD-extra (Gryfskand) was used in the study as an adsorbent, which is obtained from coal dust and binder by ironed cylindrical granules and subsequent drying, carbonization and steam activation. It is used for treatment of drinking water in large water treatment plants and small filters and containers [10].

2.2. Course of the Experiment

2.2.1. Adsorption on Virgin Activated Carbon

The virgin commercial activated carbon WD-extra (0.2 g, 0.5 g, 1 g, 1.5 g, 2 g) was placed in a conical flask with a capacity of 300 cm^3. Then 100 cm^3 of the solution of the dye was added to the flask and its concentration was 400 mg /l. The sample was shaken for 10 hours, after which time the phases (dye and sorbent) were separated. Then, the used carbon sorbent was washed with distilled water and dried in an oven.

2.2.2. Activated Carbon Regeneration

Spent activated carbon WD-extra in adsorption process has undergone regeneration using Fenton's reagent ($FeSO_4$ value: H_2O_2 ratio of 5:1). Carbon sorbent was treated with prepared in the above manner Fenton reagent solution (500 cm^3) and was shaken during different times (30 min, 1 h, 2 h, 3 h, 5 h, 8 h). Regenerated activated carbon was again used to carry out the adsorption of dyes on the regenerated activated carbon WD-extra.

2.2.3. Determination of the Concentration of Oxygen Functional Groups on the Surface of the Activated Carbon by Boehm Method

Method for characterization of the surface acidity of carbon sorbents is the Boehm method [4]. This method involves the reaction of the exchange of H^+ for Na^+ ions from the principles of various pK. The various acid groups are neutralized due to the reagents listed below [10]:

- carboxylic - under the influence of $NaHCO_3$ (0.1 mol/dm^3)
- carboxylic + lactone - under the influence of Na_2CO_3 (0.05 mol/dm^3)
- carboxylic + lactone + phenol - with NaOH (0.1 mol/dm^3)
- carboxylic + lactone + phenol + carbonyl - under the influence C_2H_5ONa (0.1 mol/dm^3)

Activated carbon was weighed of 0.2 g, was placed in a flask 100 cm^3 and then treated with 25 cm^3 of the prepared solutions. The samples were shaken for 48 hours. After this time the solution was filtered, the filtrate was collected and 10 cm^3 of the solution was titrated with HCl (0.05 mol/dm^3) against methyl orange. For C_2H_5ONa filtrate was poured into flasks containing distilled water and titrated with HCl (0.05 mol/dm^3) in the presence of methyl orange. The concentration of functional groups on the surface of the activated carbon- WD-extra was calculated by (1):

$$A = \frac{(V_{sl} - V) \bullet M \bullet 2{,}5}{m} \qquad (1)$$

where:

V_{sl} - the volume of HCl consumed for titration of 10 cm^3 of the solution in the so-called output blind sample [dm^3]

V - volume of HCl consumed for titration of 10 cm^3 of the solution after neutralization [dm^3]

M - HCl concentration [mol/dm^3]
m - mass of the activated carbon weighed sample [g]
A - amount of acid groups [mmol/g]

2.2.4. Determination of the Concentration of a Summary of a Basic Group

Activated carbon was weighted 0.2 g, was placed in a flask 100 cm^3 and then treated with 25 cm^3 of 0.05 M HCl. The solutions were shaken for 48 hours. After this time the solution was filtered, the filtrate was collected and 10 cm^3 of the solution was titrated with 0.05 M NaOH to methyl orange. The concentration of basic functional groups on the surface of activated carbon was determined by (2):

$$A = \frac{(V_{sl} - V) \bullet M \bullet 2{,}5}{m} \qquad (2)$$

where:
V_{sl} - the volume of NaOH consumed for titration of 10 cm^3 of the solution in the so-called blind sample [dm^3]
V - volume of NaOH consumed for titration of 10 cm^3 of the solution after neutralization [dm^3]
M - NaOH concentration [mol/dm^3]
m - mass of the activated carbon weighed sample [g]
A - amount od basic groups [mmol/g]

3. Results

The concentration of acidic and basic functional groups on the surface of the activated carbon was calculated by Boehm method in the experiment (Tab.1).

Functional group	Kind of sorbent						
	Virgin WD-extra	after 30 min regeneration	after 1h regeneration	after 2 h regeneration	after 3 h regeneration	after 5 h regeneration	after 8 h regeneration
Carboxylic	none	0.125	0.1875	0.500	0.625	0.9375	none
Lactone	none	0.1875	none	none	none	none	0.1875
Phenol	0.625	0.5625	0.9375	0.625	0.625	none	0.4375
Carbonyl	1.125	1.0937	4.031	3.031	3.401	2.6565	2.15625
Sum of acidic groups	1.75	1.96875	5.156	4.156	4.656	3.594	2.78125
Sum of basic groups	2.5625	2.5	2.25	1.3125	1.25	0.375	0.375

Tab.1. The concentration of surface acidic and basic groups containing oxygen on the active carbon regenerated at different times

According to the experimental results it can be clearly concluded that regeneration of the Fenton reagent affects the surface functional groups. With the recovery time increased content of acid groups and decreased content of basic groups. Amount of acid groups increased to 2.78 mmol/g, and the base has decreased to 0.375 mmol/g for carbon regenerated during eight hours.

4. Conclusion

Based on model studies it can be concluded that Fenton's reagent is an effective way to regenerate the activated carbon used in adsorption of synthetic dyes from aqueous solutions. However it has an ability to influence the oxidative surface functional groups of the activated

carbon. With the regeneration time (from 30 min to 480 min), increase in the number of acid groups, while decreasing the number of basic groups.

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Trefftz Functions for One-dimensional Non-Fourier Heat Conduction Problems

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Abstract. Non-Fourier heat conduction equation is considered. Since this is a hyperbolic equation, in which there is the second derivative of the temperature with respect to time, finding a solution of any problem makes a lot of troubles. The paper presents a method that uses Trefftz functions to build approximate solutions of initial-boundary problems for the equation. Trefftz functions are derived with the use of so-called generating functions. Also a question how to use them in order to obtain an approximate solution to this problem is briefly discussed.
Keywords: non-Fourier heat conduction, Trefftz functions.

1. Introduction

Most of heat conduction problems are described and analyzed using Fourier's law. Empirical law of propagation of heat, widely used to explain the phenomenon of heat transfer, called Fourier's law, has the form

$$\mathbf{q} = -\lambda \nabla T ,$$

where $\mathbf{q} = \frac{\mathbf{\Phi}}{A}$, [W/m^2] is the density of the heat flux flowing per unit time through a unit area, $\mathbf{\Phi}$ denotes a heat flux, [W], flowing per an area A [m^2], and λ stands for heat conduction coefficient, [W/mK]. T describes the temperature, [K]. $\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$ is the nabla operator, [1/m]. $\nabla T = gradT$, $\nabla \cdot \mathbf{q} = div\mathbf{q}$, $\nabla \times \mathbf{f} = rot\mathbf{f}$, $\nabla \cdot \nabla = \nabla^2 = \Delta$ is the laplacian; bold lower case letters mean vectors and bold large letters - matrices. Fourier's law leads to an important non-stationary heat conduction equation (also called the second Fourier conduction law), namely:

$$\nabla \cdot (\lambda \nabla T) - \rho c \frac{\partial T}{\partial t} = -Q$$

$\lambda = \lambda(T)$. At constant values of λ (λ = const) the equation can be written as

$$\nabla^2 T - \frac{1}{a}\frac{\partial T}{\partial t} = -\frac{Q}{\lambda} \qquad (1)$$

with $a = \frac{\lambda}{\rho c}$ [m^2/s], and Q is a heat source. The relation (1) describes a parabolic differential equation. It means that the Fourier's law of heat conduction predicts an infinite speed of propagation for thermal signals, i.e. a behavior that contradicts Einstein´s relativity theory.

While the heat-transfer situations include extremely high temperature gradients, temperatures near absolute zero, extremely large heat fluxes, and extremely short transient duration, the heat propagation speed is finite, and the Fourier law should be modified, [1, 2 and many others]. According to the theory presented by Cattaneo and Vernotte, [3, 4], the heat flux appears only in a posterior instant, $t + \tau$. Under these conditions Fourier's law adopts the form:

$$\mathbf{q}(\mathbf{r}, t + \tau) = -\lambda \nabla T(\mathbf{r}, t)$$

where $\boldsymbol{r}$ describes the spatial coordinates of a point, and τ is the thermal relaxation characteristic time, after which the heat flow, resulting from the infinitesimal distance $\delta\boldsymbol{r}$, necessary to construct a gradient is observed. As this distance is small, one can expand the heat flux in a Taylor series around $\tau = 0$ for small τ (as it should be, because otherwise the first Fourier´s law would fail when explaining every day phenomena) obtaining, after neglecting higher order terms:

$$\mathbf{q}(\mathbf{r}, t+\tau) = \mathbf{q}(\mathbf{r}, t) + \tau \frac{\partial \mathbf{q}(\mathbf{r}, t)}{\partial t}.$$

Thus, for λ = const we can easily obtain the following equation

$$\tau \frac{\partial \mathbf{q}(\mathbf{r}, t)}{\partial t} + \mathbf{q}(\mathbf{r}, t) = -\lambda \nabla T(\mathbf{r}, t). \tag{2}$$

Here the time derivative term makes the heat propagation speed finite. Eq. (2) tells us that the heat flux does not appear instantaneously but it grows gradually with a build-up time τ. For macroscopic solids at ambient temperature this time is of the order of 10^{-11} *s*, so that for practical purposes the use of (1) is adequate, as daily experience shows [5].

Finally one obtains:

$$\nabla \cdot (\lambda \nabla T(\mathbf{r}, t)) - \tau \rho c \frac{\partial^2 T}{\partial t^2} - \rho c \frac{\partial T}{\partial t} = -Q - \tau \frac{\partial Q}{\partial t}.$$

For constant coefficient of thermal conductivity and heat source not depending on time we have

$$\nabla^2 T - \frac{1}{a}\frac{\partial T}{\partial t} - \frac{1}{u^2}\frac{\partial^2 T}{\partial t^2} = -\frac{Q}{\lambda}, \tag{3}$$

where $u = \left(\frac{a}{\tau}\right)^{\frac{1}{2}}$ is a finite speed if propagation of the thermal signal [m/s].

2. Trefftz Method

The method known as “Trefftz method” was firstly presented in 1926, [6]. In the case of any problem an approximate solution is assumed to have a form of a linear combination of functions that satisfy the governing partial linear differential equation (without sources). The functions are termed as Trefftz functions or T-functions. In the space of solutions of the considered equation they form a complete set of functions. The unknown coefficients of the linear combination are then determined basing on approximate fulfillment the boundary, initial and other conditions (for instance prescribed at chosen points inside the considered body), finally having a form of a system of algebraic equations [7]. T-functions usually are derived for differential equation in dimensionless form.

For a homogeneous equation the T-functions can be searched by using the so-called generating functions. For non-stationary one-dimensional parabolic equation of heat conduction without heat source, in dimensionless variables, in a limited area, i.e. for the equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}\ ,\ (x,t) \in \Omega \times (0,T),\ \Omega \subset R^1, \tag{4}$$

the generating function can be found with the method of variable separation

$$\Gamma_{P1}(x,t;p) = e^{px+p^2 t} = \sum_{n=0}^{\infty} v_n(x,t)\, p^n\ . \tag{5}$$

Here $p \in R$ is an arbitrary real number. Γ denotes the generating function obtained from separation of variables, whereas v are solving polynomials. The T-functions can be obtained as follows: Introducing p = 0 into (5) one obtains the first T-function, v_0 = 1. Next, differentiating (5) with

respect to p and putting then $p = 0$, one obtains $v_1 = x$. Further differentiating of (5) and adopting $p = 0$ can effectively determine further polynomials. For (4) the T-functions are known [8].

Eq. (3) without heat source can be rewritten in a dimensionless form

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial \bar{t}} + Ve^2 \frac{\partial^2 u}{\partial \bar{t}^2}\ ,\ (x,t) \in \Omega \times (0,T),\ \Omega \subset R^1, \tag{6}$$

with $\bar{t} = \frac{at}{L^2}$, where L stands for the length scale of interest and $Ve = \sqrt{\frac{\pi a}{L^2}}$ (the Vernotte number, [9]). In the further consideration the dash over the dimensionless variable $\bar{t}$ will be omitted, i.e. we will write t instead of $\bar{t}$. For (6) two generating functions can be found, namely

$$\Gamma_1\ (x,t;p) = \exp\left(px - \frac{\left(1+\sqrt{1+(2pVe)^2}\right)t}{2Ve^2} \right), \tag{7}$$

$$\Gamma_2\ (x,t;p) = \exp\left(px - \frac{\left(1-\sqrt{1+(2pVe)^2}\right)t}{2Ve^2} \right). \tag{8}$$

Here also the method of variable separation was used to find the generating functions for (6). However, in order to generate T-functions according to the procedure described above, a program Maple was used, because such procedure for more complicated equations is practically impossible to carry out “by hand”. The first 22 T- functions resulting from the generating function (7) and (8) read

$$v_{1,0} = e^{-\frac{t}{Ve^2}},\quad v_{1,1} = xe^{-\frac{t}{Ve^2}},\quad v_{1,2} = e^{-\frac{t}{Ve^2}}\left(\frac{x^2}{2} - t\right),\quad v_{1,3} = e^{-\frac{t}{Ve^2}}\left(\frac{x^3}{3!} - tx\right),$$

$$v_{1,4} = e^{-\frac{t}{Ve^2}}\left(\frac{x^4}{4!} - \frac{tx^2}{2} + Ve^2 t + \frac{t^2}{2}\right),\quad v_{1,5} = e^{-\frac{t}{Ve^2}}\left(\frac{x^5}{5!} - \frac{tx^3}{3!} + Ve^2 tx + \frac{t^2 x}{2}\right),$$

$$v_{1,6} = e^{-\frac{t}{Ve^2}}\left(\frac{x^6}{6!} - \frac{tx^4}{4!} + \frac{Ve^2 tx^2}{2} + \frac{t^2 x^2}{4} - 2Ve^4 t - Ve^2 t^2 - \frac{t^3}{6}\right),$$

$$v_{1,7} = e^{-\frac{t}{Ve^2}}\left(\frac{x^7}{7!} - \frac{tx^5}{5!} + \frac{t^2 x^3}{12} + \frac{Ve^2 tx^3}{6} - 2Ve^4 tx - Ve^2 t^2 x - \frac{t^3 x}{6}\right),$$

$$v_{1,8} = e^{-\frac{t}{Ve^2}}\left(\frac{x^8}{8!} - \frac{tx^6}{6!} + \frac{t^2 x^4}{48} + \frac{Ve^2 tx^4}{24} - \frac{t^3 x^2}{12} - \frac{Ve^2 t^2 x^2}{2} - Ve^4 tx^2 + 5Ve^6 t + \frac{5Ve^4 t^2}{2} + \frac{Ve^2 t^3}{2} + \frac{t^4}{24}\right),$$

$$v_{1,9} = e^{-\frac{t}{Ve^2}}\left(\frac{x^9}{9!} - \frac{tx^7}{7!} + \frac{t^2 x^5}{240} + \frac{Ve^2 tx^5}{120} - \frac{t^3 x^3}{36} - \frac{Ve^2 t^2 x^3}{6} - \frac{Ve^4 tx^3}{3} + 5Ve^6 tx + \frac{5Ve^4 t^2 x}{2} + \frac{Ve^2 t^3 x}{2} + \frac{t^4 x}{24}\right)$$

$$v_{1,10} = e^{-\frac{t}{Ve^2}}\left(\frac{x^{10}}{10!} - \frac{tx^8}{8!} + \frac{t^2 x^6}{1440} + \frac{Ve^2 tx^6}{720} - \frac{t^3 x^4}{144} - \frac{Ve^2 t^2 x^4}{24} - \frac{Ve^4 tx^4}{12} + \frac{t^4 x^2}{48} + \frac{Ve^2 t^3 x^2}{4} + \frac{5Ve^4 t^2 x^2}{4} + \frac{5Ve^6 tx^2}{2} - 14Ve^8 t\right.$$
$$\left. - 7Ve^6 t^2 - \frac{3Ve^4 t^3}{2} - \frac{Ve^2 t^4}{6} - \frac{t^5}{120}\right).$$

$$v_{2,0} = 1,\quad v_{2,1} = x,\quad v_{2,2} = \frac{x^2}{2!} + t,\quad v_{2,3} = \frac{x^3}{3!} + tx,\quad v_{2,4} = \frac{x^4}{4!} + \frac{tx^2}{2} - Ve^2 t + \frac{t^2}{2},$$

$$v_{2,5} = \frac{x^5}{5!} + \frac{tx^3}{3!} - Ve^2 tx + \frac{t^2 x}{2},\quad v_{2,6} = \frac{x^6}{6!} + \frac{tx^4}{4!} - \frac{Ve^2 tx^2}{2} - \frac{t^2 x^2}{4} + 2Ve^4 t - Ve^2 t^2 + \frac{t^3}{6},$$

$$v_{2,7} = \frac{x^7}{7!} + \frac{tx^5}{5!} + \frac{t^2 x^3}{12} - \frac{Ve^2 tx^3}{6} + 2Ve^4 tx - Ve^2 t^2 x + \frac{t^3 x}{6},$$

$$v_{2,8}=\frac{x^8}{8!}+\frac{tx^6}{6!}+\frac{t^2x^4}{48}-\frac{Ve^2tx^4}{24}+Ve^4tx^2-\frac{Ve^2t^2x^2}{2}+\frac{t^3x^2}{12}-5Ve^6t+\frac{5Ve^4t^2}{2}-\frac{Ve^2t^3}{2}+\frac{t^4}{24},$$

$$v_{2,9}=\frac{x^9}{9!}+\frac{tx^7}{7!}+\frac{t^2x^5}{240}-\frac{Ve^2tx^5}{120}+\frac{t^3x^3}{36}-\frac{Ve^2t^2x^3}{6}+\frac{Ve^4tx^3}{3}-5Ve^6tx+\frac{5Ve^4t^2x}{2}-\frac{Ve^2t^3x}{2}+\frac{t^4x}{24},$$

$$v_{2,10}=\frac{x^{10}}{10!}+\frac{tx^8}{8!}+\frac{t^2x^6}{1440}-\frac{Ve^2tx^6}{720}+\frac{t^3x^4}{144}-\frac{Ve^2t^2x^4}{24}+\frac{Ve^4tx^4}{12}+\frac{t^4x^2}{48}-\frac{Ve^2t^3x^2}{4}+\frac{5Ve^4t^2x^2}{4}-\frac{5Ve^6tx^2}{2}+14Ve^8t$$
$$-7Ve^6t^2+\frac{3Ve^4t^3}{2}-\frac{Ve^2t^4}{6}+\frac{t^5}{120}.$$

3. Conclusion

Using the T-functions to find an approximate solution of a problem, especially in the case of non-Fourier heat conduction equation, greatly simplifies the calculation procedure. As an example a problem considered in [9] may be considered, where a problems of a heat flux are applied normally to the upper surface of a cylinder on a part $r < R$, where R stands for a radius of the cylinder. In order to obtain an approximate form of analytical solution a long and trouble some procedure is presented on almost 4 pages. In the case of using the T-functions so approximate the analytical solution if is a linear combination of the T-functions and the coefficients of the combination can be easily obtained numerically from minimization of the functional that describes fitting of the approximate solution to the initial and boundary conditions. Usually to minimize the objective functional (and to find the coefficients of the linear combination) the least squares method is used. Then, there is no need for time variable elimination. The method leads to smooth results and the only problem may appear with too great number of the T-functions in the linear combination, because then the truncation error may disturb the results. But there is a solution of that problem – namely the calculations have to be led on symbolic language. Also the noisy input data may be used. Some remarks concerning smoothing the noisy input data with the use of T-functions may be found in the paper [10].

It is obvious that deriving the T-functions for two and three spatial variables can be obtained in a similar way.

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Individual Predictors of Assessing the Risky Situations by Professional Drivers

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Abstract. This paper's main objective is to explore the representation of individual predictors which are predicting subjectively accepted level of risk in traffic situations of professional drivers. The study presents a view of socio-demographic, cognitive and personality predictors from the point of the risk homeostasis theory developed by J. G. Wilde. Research data were gathered by Anamnesis questionnaire, selected tests of Expert System Traffic and personality questionnaire SPIDO. The situations of confrontation with hazardous risk-driving were inducted by video-based Vienna Risk-Taking Test Traffic. The research sample of this mixed methods study consisted of one hundred forty-four professional drivers. Multiple regression analysis revealed that the most significant predictor of willingness to take risks in traffic situations is the movement and time anticipation. The diagnosis of individual predictors which are predicting the safe driving has become one of the part of job recruitment whereupon in conclusion, we state some possibilities of using our results of psychologists' transport work.

Keywords: Traffic psychology. Driving. Willingness to take risks. Prediction. Transport safety.

1. Introduction

The Bratislava public transit system consists of a network of buses, trams and trolley-buses. In fact, the network is relatively dense and most lines operate between 5 am and midnight. According to transport psychologists [1, 2] the driving profession is one of the worst professions with regard to poor health, high labor turnover and professional drivers are more exposed to traffic for long hours, which may make them more prone to fatigue and it is hardly interesting that the factors of risky behavior have been the focus of attention in traffic psychological research for a long time. At this stage, the relationship between skills, behavior and accident involvement and risky driving is complex [3], and it is a challenge for the psychology to provide a better understanding of how human factors are related to driving.

Closely related to the above figures is the concept of risky driving behavior and from its early days, research into driving psychology has therefore particular attention to the factors that determine risky driving behavior [4]. In terms of wider driving behavior, many authors argue that risk behavior describes concrete behavior in risk situations in which it is possible to choose between alternative actions with a varying level of associated risk. On the other hand, the willingness to take risks defines a global style of behavior, according to which a difference between drivers can be established the willingness to seek out risk situations [5]. In addition, one of the best-known theoretical models of risky driving behavior is the risk homeostasis theory of Wilde and according to Wilde [6] the risk homeostasis theory assumes that in the face of a given objective risk individuals will differ in the subjective level of risk they are prepared to accept. The risk behavior in a driving situation results homeostatically from interplay between a subjectively accepted degree of risk and objective traffic situation and the model assumes that individuals in everyday life accept a certain degree of risk. Depending on the characteristics of a traffic situation and given a driver's target risk value, action to adjust driving behavior may take place sooner or later and if the amount of perceived danger falls below the subjectively accepted degree of risk, according to Hergovich [5] people tend to carry out risky actions or make risky decisions.

Despite what we do know, certain individual difference variables, such as risk perception, and aspects of drivers personalities have been found to be related to an increased likelihood of unsafe driving [7]. However, in recent years, the reaches have no placed more emphasis on the socio-demographic, cognitive and personality predictors related to driving among bus drivers, tram drivers or trolley-bus drivers. For instance, in the study of professional Turkeys `drivers in by Sümer [8] was found that sensation-seeking and aggression were together predictive of risky driving among non-professional drivers. In fact, Porter [1] argues that driver age, previous accidents, driving experience, work conditions, the type of vehicle (minibus, public light bus), their severity are correlated with risk of being involved in another accident. Professional drivers throughout the world have a particular demographic profile and they are involved a disproportionately high number of road crashes [2]. The main problem associated with these kinds of measures is the sample of non-professional drivers.

Giving this context, our present study therefore assessed three individual groups of predictors which are hypothesized to predict subjectively accepted level of risk in traffic situations - socio-demographic, cognitive and personality factors among Slovak professional drivers.

2. Method

2.1. Sample

The sample consisted of 144 respondents which all were men, aged between 25 and 71 years. The median age was 53.5. A total of 68 (47.2%) respondents were trolley-buses public transport drivers, 52 (36.1%) respondents was tram public transport drivers, 24 (16.7%) bus public transport drivers from the region of Bratislava. A total of 13 (9.0%) respondents had completed primary school or basic secondary school without completing vocational training, 83 (57.6%) respondents had also completed vocational training, 45 (31.3%) respondents had a qualification at university entrance level and 3 (2.1%) respondents had a university degree. Participation was voluntary.

2.2. Measures

With the view of the goal our study achieved results the respondents were assessed individually. The test battery consisted of the following tests.

2.2.1. Vienna Risk-Taking Test Traffic

This test was used to measure “subjectively accepted level of risk in traffic situations”. It is an objective standardized test that assesses risk-taking behaviour in potentially dangerous driving situations (overtaking situations, speed choice, intersections, and decisions). During the test a total of 24 videos are presented. The respondents have to state each time by pressing the key at which distance from the potential danger the represented situation becomes dangerous. Each traffic situation is shown twice: on the first occasion, respondents simply observe the situation and on the second occasion they have to make a response [9].

2.2.2. Anamnesis’ Questionnaire (AQ)

The test consisted questions which was divided into questions about socio-demographic factors such as their age, years of driving experience, type of driver (trolley-buses public transport driver, tram public transport drivers or bus public transport driver), self-committed traffic accidents within the last ten years, and not self-committed traffic accident.

2.2.3. Adaptive Matrices Test (AMT)

This test is a non-verbal test for assessing “general intelligence” as revealed in the ability to think inductively. The test was administered as a computerised adaptive test and the items resemble

classical matrices. The respondents can choose one of nine alternative answers, making his selection by means of mouse [10].

2.2.4. Adaptive Tachistoscopic Traffic Perception Test (ATAVT)

The test tests visual observational ability and skill in obtaining an overview, and speed of perception by briefly presenting pictures of traffic situations [11]. The respondent is briefly shown pictures of traffic situations and after seeing each picture, he is asked to state what was in it, choosing from five answer options that he is given.

2.2.5. Cognitron (COG)

Cognitron test was administered in order to measure “selective attention”. In the test forms with flexible working time the respondent`s task is to compare an abstract figure with model and to decide whether the two are identical. The test application is about the assessment of concentration through comparison of fig figures with regard to their congruence [12].

2.2.6. Test ZBA Time Movement Anticipation

ZBA [11] is the test of the estimation of the motion of objects in a space. This test is used to assess “anticipation of movement and time”. A green ball appears on the screen, moving slowly and at an unpredictable moment the ball disappears and two red lines appears, the respondent is asked to indicate the point at which the ball will cross the target line.

2.2.7. SPIDO

Personality traits were tested by the SPIDO questionnaire [13] with four basic factors: cognitive variability, emotional variability, self-regulative variability, adaptive variability.

3. Results

In this study, the calculations were carried out using SPSS Statistics 19.00. We began our investigation of the relationship between the variable willingness to take risks in traffic situations and the individual factors.

Number of self-committed traffic accidents is significantly correlated with willingness to take risks in traffic situations (r=.212, p<.05), but it is not correlated with not self-committed traffic accident. In our sample, willingness to take risks shows a significant positive correlation with selective attention (r=.255, p<.01) and time and movement anticipation (r=-.301, p<.01), but it is not related to general intelligence (r=-.046) and skill in obtaining an overview (r=.109). Risky driving behavior in potentially hazardous situations correlates with both self-regulative variability (r=.178, p<.01), adaptive variability (r=-.200, p<.01), but it is not correlated with cognitive variability (r=-.150) and emotional variability (r=.000). Tab. 1 shows the results of a linear regression analysis. The multivariate results were calculated by means of Forward selection of the predictor variables in the total sample (N=144). When all factors had been included at the final step of regression analysis, stepwise logistic regression leads to a model accounted for 34.6% of the explained variance. For the model of the willingness to take risks the Pearson correlation coefficient amounts R=.588.

As can be seen from Tab. 1, all seven variables retain in the model are: time and movement anticipation, selective attention - time of correct answer, self-committed traffic accidents, adaptive variability, bus driver, years of driving experience, cognitive variability.

Predictors	β	p	R	R Square (R^2)	ANOVA
ZBA: Time and movement anticipation	-.326	<.001	.588	.346	F=10.285 p=.000 p<.01
COG: Selective attention- time of correct answer	.348	<.001			
AQ: Self-committed traffic accidents	.233	p			
SPIDO: Adaptive variability	-.254	p			
AQ: Bus driver	-.177	.013			
AQ : Years of driving experience	.220	.013			
SPIDO : Cognitive variability	.188	.015			

Tab. 1. Regression analyses summary, illustrating the factors significantly predicting of willingness to take risks in in traffic situations.

The variable movement and time anticipation obtained from the ZBA test is into the regression model first (β=-.326, p=.000). The standardized β weights indicate that a lower subjectively accepted level of risk in traffic situations (lower raw scores) is associated with a better ability of anticipation movement and time (higher raw scores).

The results for the overall model demonstrated that the variables Selective attention-time of correct answer (β=.348, p=.001), Self-committed traffic accidents (β=.233, p=.001), Years of driving experience (β=.220, p=.013) and Cognitive variability (β=.188, p=.015) are significant positive predictors for the willingness to take risks in traffic situations. In addition, seeking Time and movement anticipation (β=-.326, p=.000), Adaptive variability (β=-.254, p=.000), Bus driver (β=-.177, p=.013) were a significant negative predictors.

4. Discussion and Conclusions

A main aim of this study is to demonstrate that individually factors contribute to risky driving behaviors in the same way. On a basis of the theory of risk homeostasis developed by Wilde, we tested individual predictors that were hypothesized to predict the risky driving behavior prior to any other predictor. The individual data in this study are consistent with the available literature, but they also provide some interesting and new insights.

In total, the most important paper's conclusion that can be drawn from this study is that the estimation of the motion of objects in a space predicted subjectively accepted level of risk in potentially hazardous driving situations. This finding indicates that the professional driver with this impaired ability has a greater tendency to take a higher level of risks. A possible explanation for this finding is poor anticipation of future considering the consequences of the current traffic situation. These results are harmonious with Zimbardo's et al. and A. Cheng's [14, 15].

Interestingly, some relationships between cognitive factors and risky driving were also observed. The driver's well ability, even when under time pressure, to focus his attention on relevant matters and is thus able to work accurately to significantly influence the tendency to take risks. The drivers who have had tendency to search for more stable interactions they have difficulty in finding an appropriate speed for correct decision. They are distinguished from those who have had tendency to search for change and variability of external intensive simulation in their perception which is entirely in line with the conclusion of Groeger [16]. Our result suggests that drivers with a high level of subjectively accepted risk tend to accept a greater degree of objective danger and they cause more number of road accidents than individuals with a low level of subjectively accepted risk. This interpretation is supported by the results obtained in a study conducted by Knipling [17]. To a lesser extent, the adaptive variability concerning the driver's tendency to adaptive behavior into new environment was also associated with the subjectively accepted level of risk in traffic situations.

Another fascinating finding is the relationship between willingness to take risks in traffic situations and type of professional driver – trolley-buses, tram and bus public transport driver. Specifically, in our sample, bus drivers tend to accept a greater degree of objective danger than trolley-buses and tram drivers. A possible explanation for this finding might be that the bus drivers

of public transport are able to more free style of driving. We point out that the tram is a vehicle which runs on fixed rails and the trolley-buses is a wheeled vehicle that runs on rails and. In these cases, both vehicles are in some way restricted by specific a movement, in contrast buses. We think that the willingness to take risks could be also determinate by professional career in buses driving, who is profiled over the years the driving of certain types of vehicles. A study by Oza also provided that the minibus drivers tend to be more aggressive than private drivers and they are usually exposed to more difficulties and stressed caused by traffic [18]. In line with previous studies by Castella [19] and Zuckerman [20], inexperienced drivers who tended to be involved in higher levels of speeding during their driving also overestimated their level of skill as drivers and underestimated the potential risk of traffic driving situations. Furthermore, to being driving experienced associated with higher willingness to take traffic risks among professional drivers, which provides useful information for road safety interventions and the development of driver training programs.

To summarize, in practical applications presented in this paper is that the predicted willingness to take risks in traffic situations can be used in a standardized assessment centre to pre-screen drivers who want to go to driving tests for the position of professional trolley-buses public transport drivers, tram public transport drivers, bus public transport drivers. Our study lend preliminary support to driver training that more attention must be directed toward education of professional drivers and strategies to deal with the hazardous situations in traffic system.

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Survey Records in the Recovery of Expropriated and Nationalised Properties in Poland

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Abstract. This publication is devoted to the issue of legal constructs enabling the restitution of expropriated property and the recovery of nationalised property in Poland. It emphasises the aspect of survey records prepared by a licensed surveyor for the needs of administrative and court proceedings concerning the pursuit of claims for the restitution of property as well as presents the role of a property appraiser as exemplified by the appraisal of properties taken over illegally. It submits for consideration selected activities performed by a surveyor and selected problems he encounters while working with documentation specifying the border line of a property being the object of expropriation or nationalisation respectively. These difficulties result, in particular, from the changes which over the years took place in the structure of boundaries, ownership relations and in the manner of property boundaries registration.

Keywords: Expropriation, recovery, nationalization.

1. Introduction

Ownership is the strongest property right in Poland and its protection is currently guaranteed by the Constitution of the Republic of Poland. There are, however, legal instruments enabling the acquisition of rights by the state to a property to which another entity is entitled, including expropriation and nationalisation. Expropriation is currently a basic legal construct in the process of acquiring by the State Treasury or local government units rights to a property to which other entities are entitled and which are indispensable for public purposes. Another manner of property acquisition, but solely for the State, is nationalisation. The legal instruments which will be described below enable the restitution of expropriated or nationalised property. Special attention will be paid to the aspect of survey records prepared for the needs of administrative and court proceedings connected with them.

The scale of claims for restitution of property in Poland is reflected by, among other things, the data given by the Agricultural Property Agency executing ownership rights and other property rights towards agricultural properties of the State Treasury. As of 30th November 2008, 3081 claim applications of former owners or their legal successors were received by the APA. They concerned about 600 thousand ha, that is about 25% of the area of land remaining in the Resource, whereas according to the data compiled at the end of 2008, reprivatisation claims were raised towards 200 out of 825 manor complexes and park-and-palace complexes remaining in the Resource. The scale of the problem of claims concerning land parcelled out in allotment gardens was brought to attention by "the announcement on the claimed allotment gardens as of 1st July 2012." It says that claims are laid to 342 allotment gardens and the area of 1373,9002 ha (33073 allotment gardens). It is, however, only a part of the claimed lands.

2. Restitution of Expropriated Property

The Real Estate Management Act, which governs the principles of property expropriation, regulates the possibility of its restitution, that is, restoring the legal status prior to issuing the decision on expropriation. The previous owner or his heir may demand the restitution of an expropriated property if it has become redundant for the purpose specified in the expropriation decision. Such a situation takes place when, in spite of the lapse of 7 years since the day on which the decision became final, work connected with the accomplishment of the goal has not begun, or when the goal has not been achieved despite the lapse of 10 years.

A body is obliged to notify the previous owner or his heir of the intention to use the expropriated property or its part for the purpose different than the one specified in the expropriation decision and then within 3 months from the day of receiving the notice, the person has the right to file a restitution application. If the person does not submit the application on time, this entitlement expires.

Art.216 additionally extends the scope of property restitution possibilities in the case of properties acquired by the State Treasury by referring to other legal acts constituting the grounds for restitution claims. These properties which have been disposed in favour of the third party are not subject to property restitution. The elimination of the effects of final expropriation decision may occur also in the cases defined in the Administrative Procedure Code, for instance, by way of the declaration of invalidity of the decision and the resumption of the proceedings in case of decision defects or the defects of the proceedings preceding its issuing.

3. The Restitution of Nationalised Property

Another manner of property acquisition is nationalisation. The transition of ownership takes place by virtue of a normative act which is usually equivalent to an act of Parliament. Nationalisation involves a certain category of estates and rights (eg. banks, enterprises, etc.) Nationalisation acts have to include not only the decision on nationalisation but also the determination of its mode [Osajda, 2009]. An adjudication, which constitutes the grounds for the entry in land and mortgage registers, is frequently necessary for nationalisation to produce legal effects. Examples of nationalisation encompass the state takeover of land properties, abandoned properties and properties abandoned by German refugees, forests, Warsaw lands, etc., which took place after World War II. An increased interest in the restitution of property of former owners and their heirs occurred together with the changes in the socio-economic system. Because of the lack of statutory provisions on reprivatisation, the restitution of nationalised property may be conducted in individual cases by way of administrative and court proceedings. The aforementioned institution of property restitution does not give the possibility of the recovery of the property nationalised by virtue of administrative acts other than an expropriation decision or general acts not specified in art. 216 of the Real Estate Management Act. Court reprivatisation applies to these cases in which the authorities infringed the provisions of law and its scope is limited.

4. The Role of a Surveyor and a Property Appraiser in Restitution Proceedings Concerning Expropriated and Nationalised Property

Both administrative and court proceedings entail the demand for geodetic documentation prepared by a licensed surveyor. It is necessary to determine the boundaries of a property being the object of expropriation or nationalisation. This task may pose a difficulty, taking into consideration the changes which over the years took place in the structure of boundaries, ownership relations and in the manner of property boundaries registration. For the needs of survey records, a surveyor often carries out an analysis of materials prepared at different times, often returning to materials dating

back to the period of the Partitions of Poland when in Poland different metric systems, nine systems of triangular networks referred to the level of three seas (the Baltic Sea, the North Sea and the Adriatic Sea), two cadastral systems (Prussian and Austrian) and the lack of cadastre in the eastern area of the former Russian sector were in operation. The registration of factual circumstances, by virtue of the decree of 1955, which had been taking place until the Act of 1989 "Geodesy and Cartography Law" came into force, is of no small importance either. According to the parcel definition of the time, a parcel was supposed to constitute a homogeneous area of possession and not of freehold as previously. The process of registering the state of possession was independent of the operation of land and mortgage registers being evidence of freehold whereas the already mentioned Act of 1989 "Geodesy and Cartography Law" introduced the obligation of disclosing owners in land and mortgage registers. The Constitution of the Republic of Poland of 1997 also emphasised the significance of ownership, stressing that the Republic protects ownership (art.21).

Preparing surveying documentation for the needs of the above administrative and court proceedings, a surveyor is supposed to, among other things, eliminate the discrepancies between cadastral data and data inscribed in the land registry. Since cadastral revision resulted in numerous changes in the numbering and area of parcels, synchronisation lists are an essential element of today's survey records prepared for restitution purposes. Synchronisation lists help to indentify which present parcel corresponds to an expropriated or nationalised parcel. Modern computer techniques of constructing the layers of vector and raster maps facilitate the investigation of property boundaries [Dzikowska, 2010], however, the possibility of reconstructing the boundary lines depends largely on the availability and state of documentation constituting the basis for a property legal description. If in the state geodetic and cartographic resource, there are records including cadastre data (land and mortgage registers) forming the basis for a description of a property in the land and mortgage register and they contain numerical data specifying the location of boundary monuments and the boundary lines of a property, then the coordinates transformation of border monuments into the current system of coordinates is performed and the previous boundary structure, reconstructed on the basis of transformation, is put on the structure existing in the current database. There are not infrequent situations, however, in which there are no numerical data to reconstruct the extent of the property ownership and the boundary line should be determined on the basis of maps and plans included in the document collections of land and mortgage registers. It is particularly difficult in the case of analysing the content of old land registries. Old land registries, according to the ordidnance of the Minister of Justice of 14th July 1986, are registries which as of 1st January 1989 ceased to be effective and were subject to closing ex officio if they did not contain in their second, third and fourth sections any entries made after 31st December 1946. They still remain documents and a valuable source of information on the boundary lines of parcels, especially during the preparation of surveying documentation for legal status regulation for the needs of nationalised property restitution proceedings. Unfortunately not all prewar land and mortgage registers survived warfare. In some cases, although registers survived, maps and plans forming the basis for property legal description were destroyed or lost, which sometimes makes it impossible to reconstruct the boundary lines

Surveying documentation for the needs of restitution proceedings frequently includes a land division project. If, for instance, the purpose of expropriation has been accomplished only on the part of an expropriated property, its former owner is entitled to the possibility of demanding the restitution of the remaining part. In such a case a licensed surveyor prepares a map with a division project, taking into consideration the recommendations of the body concerning new boundary lines which reflect the limit of the property redundancy for the purpose specified in the expropriation decision. Such division may be performed independently of the findings of the local plan or the decision on land development and management conditions. Therefore, it does not require issuing divisory permission by the commune head (wojt), the mayor or the mayor of the city. In the case of sectioning off a part of property for restitution, the decision approving the division is not issued either because it is approved by the final decision on expropriated property restitution.

Another aspect of administrative and court proceedings concerning property restitution are appraisal reports. The appraisal of losses incurred in the post-war period and resulting from, often illegal, nationalisation is both particularly interesting and difficult. Former owners whose properties were taken over with law infringement are entitled together with their heirs and devisees to the indemnification of the damage in kind. However, there is usually no possibility of restitution due to irreversible legal effects which occured in the meantime. In such cases aggrieved persons are entitled to pecuniary compensation. The present provisions of law on property appraisal do not contain explicit rules of determining the parameters of appraisal of properties taken over illegally. Therefore, it is a court appointed property appraiser's responsibility to establish appropriate procedures for each individual appraisal purpose. The most important element of the entire process is its accurate examination which determines the choice of a suitable appraisal procedure. Establishing the amount of compensation for lost properties, experienced property appraisers suggest considering appraisal parameters for similar purposes, that is take into account the state of property as of the day of ownership loss and the price of property as of the day of appraisal.

5. Conclusion

The interpretation of the extent of property ownership both expropriated and nationalised plays a crucial role in property restitution proceedings. In order to properly interpret materials collected in the state geodetic and cartographic resource, state archives and district courts, today's surveyor has to be knowledgeable about the present and historical aspects of the operation of cadastre in Poland as well as basic issues concerning legal constructs enabling the restitution of expropriated property and the recovery of nationalised property.

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Linear Mixed Model for Analysis of Biomedical Experiments

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Abstract. The linear mixed models have been used for long time in different areas of biomedical and technical research. The main aim of this paper is to provide a brief overview of the theory about these models and an application of this theory to analysis of biomedical data. Analysis of repeated measures data is realized by statistical computing software SAS/STAT (procedure MIXED).

Keywords: Linear mixed model, fixed effects, random effects.

1. Introduction

Linear mixed model has the following form:

$$y = X\beta + Zu + e \tag{1}$$

where y is the n-dimensional column vector of observed data, β is the unknown p-dimensional column vector of fixed effects, u is the unknown r-dimensional column vector of random effects with $E(u)=0$ and $Var(u)=G$, e is the unknown n-dimensional column vector of random errors with $E(e)=0$ and $Var(e)=R$, and consequently, $E(y)=X\beta$ and $Var(y)=V=ZGZ'+R$. The variance-covariance matrices G and R are typically parameterized by an s-dimensional $(s<<n)$ vector of variance-covariance components, say θ, so if necessary to emphasize the dependence on the variance-covariance components we shall write $G(\theta)$, $R(\theta)$ and $V(\theta)$, respectively. The $(n\times p)$-matrix X and the $(n\times r)$-matrix Z are the known design matrices, the matrix X is the matrix of constants that describe the study's structure with regard to the fixed effects and the matrix Z is the matrix of constants that describe the structure of the study with respect to the random effects.

Thus, in the linear mixed model we distinguish two kinds of effects: fixed effects and random effects and the name "mixed" comes from this fact. An effect is called fixed if the levels in the study represent all possible levels of the factor, or at least all levels about which inference is to be made. An effect is called random if the levels in the study represent only a random sample of a larger set of potential levels of the factor. The assumptions about random effects are different from those for fixed effects and so treatment of the two kinds of effects is not the same. A fixed effect is considered to be a constant, which we wish to estimate, but a random effect is not considered to be a constant and we want to not only to predict the random effect, but also to estimate their variances.

2. Estimating Fixed Effects and Predicting Random Effects

We need to estimate the fixed effects β and predict the random effects u. First, let us consider that G and R are known variance-covariance matrices. Then the joint probability density function of the vector (y, u) is given as

$$f(y,u) = \frac{1}{(2\pi)^{(n+r)/2}} \begin{vmatrix} G & 0 \\ 0 & R \end{vmatrix}^{-1/2} \exp\left\{ -\frac{1}{2} \begin{bmatrix} u \\ y - X\beta - Zu \end{bmatrix}' \begin{bmatrix} G^{-1} & 0 \\ 0 & R^{-1} \end{bmatrix} \begin{bmatrix} u \\ y - X\beta - Zu \end{bmatrix} \right\}. \tag{2}$$

We set the partial derivatives of this function equal to zero, i.e.

$$\frac{\partial f(y,u)}{\partial \beta} = 0, \qquad \frac{\partial f(y,u)}{\partial u} = 0 \tag{3}$$

and this leads to the equations in the following form

$$\begin{bmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z + G^{-1} \end{bmatrix} \begin{bmatrix} \tilde{\beta} \\ \tilde{u} \end{bmatrix} = \begin{bmatrix} X'R^{-1}y \\ Z'R^{-1}y \end{bmatrix}. \tag{4}$$

The equations (4) were developed by Henderson in [1] and they are known as the mixed model equations (MMEs). The solutions are

$$\begin{bmatrix} \tilde{\beta} \\ \tilde{u} \end{bmatrix} = \begin{bmatrix} (X'V^{-1}X)^{-1}X'V^{-1}y \\ GZ'V^{-1}\left(y - X(X'V^{-1}X)^{-1}X'V^{-1}y\right) \end{bmatrix}, \tag{5}$$

provided that the inverse matrix $(X'V^{-1}X)^{-1}$ does exist. The MME solution $\tilde{\beta}$ is the BLUE (best linear unbiased estimator) of the fixed effects β and $\tilde{u}$ is the BLUP (best linear unbiased predictor) of the random effects u. For more details see e.g. [7].

In the case that the variance-covariance matrices $G(\theta)$ and $R(\theta)$ are unknown (i.e. the vector of variance-covariance components θ is unknown, however the structure of $G(\theta)$ and $R(\theta)$ is assumed to be known), we should estimate the variance-covariance components θ either by the ML (maximum likelihood) method or the REML (restricted or residual maximum likelihood) method. The corresponding log-likelihood functions are as follows:

$$\begin{aligned} ML: \quad & l(\beta,\theta \mid y) = -\frac{1}{2}\ln|V(\theta)| - \frac{1}{2}(y - X\beta)'V(\theta)^{-1}(y - X\beta) - \frac{n}{2}\ln(2\pi) \\ REML: \quad & l_R(\theta \mid y) = -\frac{1}{2}\ln|V(\theta)| - \frac{1}{2}\ln\left|X'V(\theta)^{-1}X\right| - \frac{1}{2}r'V(\theta)^{-1}r - \frac{n-p}{2}\ln(2\pi) \end{aligned} \tag{6}$$

where p is the rank of X and

$$r = y - X\left(X'V(\theta)^{-1}X\right)^{-}X'V(\theta)^{-1}y, \qquad V(\theta) = ZG(\theta)Z' + R(\theta). \tag{7}$$

Let $\hat{\theta}$ denotes the estimator of θ (e.g. the REML of θ), and further let $\hat{G} = G(\hat{\theta})$, $\hat{R} = R(\hat{\theta})$ and $\hat{V} = V(\hat{\theta})$. If we formally solve the MMEs (4) by using $\hat{G}$ and $\hat{R}$ instead of G and R, we get the solutions $\hat{\beta}$ and $\hat{u}$ which are different from the $\tilde{\beta}$ and $\tilde{u}$, and have no more the optimality properties of the BLUE and the BLUP, respectively. They are typically denoted as EBLUE (empirical BLUE) of β and EBLUP (empirical BLUP) of u. The statistical inference on the linear functions of fixed and random effects, i.e. $w=K'\beta+L'u$, based on the EBLUE or the EBLUP, is a problem of fundamental interest for data analysis by linear mixed models. For more details see e.g. [7].

3. Example

First, we consider application of a simple linear mixed model with two variance components estimated by REML. The following data in the Tab.1. are from [6] and represent growth measurements for 11 girls and 16 boys at ages 8, 10, 12, 14.

Person	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Age\Gender	F	F	F	F	F	F	F	F	F	F	F	M	M	M
8	21.0	21.0	20.5	23.5	21.5	20.0	21.5	23.0	20.0	16.5	24.5	26.0	21.5	23.0
10	20.0	21.5	24.0	24.5	23.0	21.0	22.5	23.0	21.0	19.0	25.0	25.0	22.5	22.5
12	21.5	24.0	24.5	25.0	22.5	21.0	23.0	23.5	22.0	19.0	28.0	29.0	23.0	24.0
14	23.0	25.5	26.0	26.5	23.5	22.5	25.0	24.0	21.5	19.5	28.0	31.0	26.5	27.5

Person	15	16	17	18	19	20	21	22	23	24	25	26	27
Age\Gender	M	M	M	M	M	M	M	M	M	M	M	M	M
8	25.5	20.0	24.5	22.0	24.0	23.0	27.5	23.0	21.5	17.0	22.5	23.0	22.0
10	27.5	23.5	25.5	22.0	21.5	20.5	28.0	23.0	23.5	24.5	25.5	24.5	21.5
12	26.5	22.5	27.0	24.5	24.5	31.0	31.0	23.5	24.0	26.0	25.5	26.0	23.5
14	27.0	26.0	28.5	26.5	25.5	26.0	31.5	25.0	28.0	29.5	26.0	30.0	25.0

Tab. 1. Growth measurements.

The SAS code for these data is given by

```
data pr;
  input Person Gender $ y1 y2 y3 y4;
  y=y1; Age=8;   output;
  y=y2; Age=10; output;
  y=y3; Age=12; output;
  y=y4; Age=14; output;
  drop y1-y4;
  datalines;
 1   F   21.0   20.0   21.5   23.0
…….……..……..……..……..……..……..…
27   M   22.0   21.5   23.5   25.0
;
```

Here the total number of observations is $n=108$, the matrix X is $(n\times p)$-dimensional with $p=6$ and with $rank(X)=4$, and the vector of fixed effects is $\beta=(\beta_1,\ldots, \beta_6)'$. The matrix Z is $(n\times r)$-dimensional with $r=27$ and the vector of random effects is $u=(u_1,\ldots,u_{27})'$ with $E(u)=0$, $Var(u)=G(\theta)=\sigma_1^2 I_{27}$ and $e=(e_1,\ldots,e_{108})'$ with $E(e)=0$, $Var(e)=R(\theta)=\sigma^2 I_{108}$. Here $\theta=(\sigma_1^2,\sigma^2)$ denotes the vector of the unknown variance-covariance parameters. The SAS code for estimating the fixed effect parameters and the variance-covariance components as well as for testing statistical significance of the fixed effects is given by (pr is the name of the dataset)

```
proc mixed data=pr method=reml covtest;
  class Person Gender;
  model y = Gender Age Age*Gender / E3 DDFM=KENWARDROGER s;
  random int / subject=Person;
run;
```

The results are in Tab.2. The estimate of the boys' intercept is 16.3406 and the estimate for the boys' slope is 0.7844. The estimate of the girls' intercept is 17.3727 and the estimate for the girls' slope is 0.4796. From these results we can draw conclusions that the girls' starting point is larger than that for the boys, but their growth rate is smaller than boys' growth rate.

Effect	Gender	Estimate	Standard Error	DF	t Value	Pr>|t|
Intercept		16.3406	0.9813	104	16.65	<.0001
Gender	F	1.0321	1.5374	104	0.67	0.5035
Gender	M	0	.	.	.	.
Age		0.7844	0.07750	79	10.12	<.0001
Age*Gender	F	-0.3048	0.1214	79	-2.51	0.0141
Age*Gender	M	0	.	.	.	.

Tab. 2. Solution for fixed effects.

The option DDFM=KR requests that degrees of freedom for use F-statistics be estimated by the Kenward and Roger's method [3].

To follow Jennrich and Schluchter, the next example uses different version of LMM and the ML method instead of the default REML method to estimate the unknown covariance parameters. The model for fixed effects (i.e. the matrix X and the vector β) is the same, but the covariance structure is more complicated, modeled only by matrix $V(\theta)=R(\theta)=diag(Ri)$, where $R_i(\theta)$ for each

$i=1,\ldots,27$ is the same (4×4) matrix with unknown variance-covariance components (because R_i is symmetric it depends only on 10 unknown parameters $\theta=(\theta_1,\ldots,\theta_{10})$). The SAS code is given by

```
proc mixed data=pr method=ml covtest;
  class Person Gender;
  model y = Gender Age Gender*Age / E3 DDFM=KENWARDROGER s;
  repeated / type=un subject=Person r;
run;
```

The results for fixed effects are in Tab.3. The estimate of the boys' intercept is 15.8423 and slope is 0.8268. The estimate of the girls' intercept is 17.4254 and slope is 0.4764.

Effect	Gender	Estimate	Standard Error	DF	t Value	Pr>|t|
Intercept		15.8423	1.0063	27	15.74	<.0001
Gender	F	1.5831	1.5765	27	1.00	0.3242
Gender	M	0	.	.	.	.
Age		0.8268	0.08509	27	9.72	<.0001
Age*Gender	F	-0.3504	0.1333	27	-2.63	0.0140
Age*Gender	M	0	.	.	.	.

Tab. 3. Solution for fixed effects.

We can see that these results are slightly different from the previous results. In general, it is important to select a reasonable method and covariance structure in order to obtain valid inferences for fixed effects, because different kinds of the method and covariance structure give different results.

4. Conclusion

Here we have provided a brief overview of the theory about linear mixed models and an application of this theory to analysis of biomedical data. The reason why these models are often preferred is that they are able to take into account various sources of variability. Although the linear mixed models have been used for long time in different areas, there are still some open problems, for example how to approximate the test statistics distribution (e.g. by Kenward-Roger method) and subsequently the study of the statistical properties of these approximation.

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Several Notes on (Non)Existence of Absolute Synonymy

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Abstract. Synonymy is one of the typical features of the lexicology itself. However the main aim of this paper is to point out the existence or nonexistence of the so called absolute synonymy that represents one peculiar kind of synonymy. It examines this contentious issue starting from the providing of basic definitions through searching for main characteristics of these features and leading to presenting and analysing linguists´ opinions. The secondary aim of this paper is to clarify how some things, appearing also in the field of lexicology, that seem to be quite simple could be in fact really complicated and difficult to solve.

Keywords: synonymy, absolute synonymy, existence, nonexistence.

1. Introduction

One of the most discussed topics in lexicology is the one dealing with problems of existence or nonexistence of absolute or strict synonymy. Frequently asked question concerning this linguistic phenomenon can be stated as follows: Can we find absolute synonymy in any language? Seemingly, it is really easy to provide answer to this question. However, going through several materials dealing with absolute synonymy and synonymy at all, shows us that this is the case where things are not as easy as they seem to be at first sight. It also deals a little bit with philosophy because many items could be examined here from very different points of view and it is not possible to say which one should be regarded as the one that should be followed. Therefore it is necessary to start with an attempt to define the term "synonymy".

2. Synonymy

1.1. Main Characteristics of Synonymy

It is common that opinions of many linguists about definitions of synonymy vary. The basic one, which is taught at primary as well as secondary schools, sounds like: synonyms are words, which have identical meanings but they differ in their forms (or formal representation). Another one is: when two words have the same meanings, we say that they are synonymous. It is general simplification to define synonymy as sameness. Some linguists state that synonyms are words whose similarities are greater than their differences. This definition is also not exact enough. Our communication is made possible not by similarities, but just by differences (in the field of each linguistic discipline) among single words. According to Oxford Dictionary "synonym is a word or expression that has the same or nearly the same meaning as another in the same language" [5]. However, e.g. if we translate something, we translate meaning of the words, so it could be said that at that time we are looking for synonyms or synonymous expressions in our mind to preserve the most of given information. Thus synonymy is not possible only within one language but also across many different ones.

It might be impossible to find out one definition which all linguists would agree on, but in spite of this fact, there are still two groups of linguists. One group regards synonymy as sameness of meaning, the other one as similarity of meaning and therefore if we speak about synonymy in general it is clear that we deal with "near synonymy" because what could be said is exactly the

same as something else (considering the absolute sameness) and that there is simply no difference. Nothing. It does not depend on whether we take e.g. two photos of the same man or twins, there will be always something different, maybe not anything like permanent characteristic, but at least in that particular moment, as Heracleitos said: "You never step twice into the same river". It is not possible to say about one thing that it is completely the same in each aspect as anything else. Therefore a definition of synonyms which we consider the most accurate, describes synonyms as "words only similar in meaning, not identical and interchangeable only in certain contexts" [3].

Linguists have never been satisfied with one homogeneous group, so they try to classify synonyms into the so called categories. The result is that we can divide them into "subclasses" such as expressive synonymy, stylistic synonymy, duplicates (variants), absolute synonymy, prepositional synonymy, ideographic synonymy and many others. It depends on the criteria, e.g. degree, different shades of meaning and emotion, different attitude and grammatical pattern, etc.

1.2. Absolute Synonymy

In this paper the focus is just on phenomenon of "absolute synonymy". Does it exist or not? If we want to start looking for the answer it is necessary to know what it means. From the aforementioned, it is more or less clear what covers the term of synonymy, but it is not only synonymy but ABSOLUTE synonymy. It logically indicates that there have to be met some more additional conditions for two words to be ABSOLUTELY synonymous. We have found out that opinions of linguists differ and up to now there is not just one generally accepted or better called a universal definition (theorem) what something like absolute synonymy is. Therefore we decided to continue in certainly unexpected way.

It should not be right to list here certain criteria, and claim that if these all are not met there is no absolute synonymy between these words. Many reasons could be stated to prove that this is not the right way; it would still be only one of many attempts with impossibility to give the FINITE number of rules for absolute synonymy to be identified. For this reason, this will be an attempt to clarify everything known what is understood under absolute synonymy by looking for the answer on the most discussed question related to this topic: Does absolute synonymy exist or not? We will examine both possible viewpoints and the reasons which are used to support their arguments.

Many linguists claim that there is no such thing as absolute synonymy. This "perfect equivalence" does not exist in any language. Each word has got a set of associations or components unique to it. Although many words share some of these associations if they shared all of them, the second one would not be needed and would not have existed. According to Abu Hila:l Al-'Askari it would be a pointless linguistic duplication; two words with the same meaning. Cruse thinks only about similarity with the connection to synonyms and any dictionary of synonyms confirms his opinion. There are differences to be found among different dictionaries, "some lexical items that are taken to be synonymous are not really very much so", e.g. the Dictionary of English synonyms stated kill as a synonym of murder (but not vice versa), strong as synonym of powerful, but there is absence of cognitive synonymy, strong car is not the same as powerful car and accidental killing is not murder. Cruse also adds that some pairs of synonyms are "more synonymous" than others e.g. settee and sofa are more synonymous than die and kick the bucket and they are more synonymous than boundary and frontier, etc. If he thinks about certain scale of synonymy where is an exact borderline between synonymy and nonsynonymy?

The presented examples showed that there is a borderline in each individual human brain because it is just the human brain that decodes messages in the act of communication. We express our thoughts, opinions, inner thinking, and emotions by words. Sometimes we feel that we can find many words which are suitable, sometimes only one and there are also cases when we are unable to find certain word to express something because there is simply not such word which would have that particular meaning. According to Nida, it is clear that meanings of synonymous items overlap and thus they are substitutable one for another at least in certain contexts, "but rarely, if ever, are two terms substitutable for each other in any and all contexts" [4]. Sometimes when e.g. lexical

units are not in one's vocabulary it is not possible for that person to determine how such meanings differ from one another and although their related meanings seem to be so close to each other that one is not able to determine whether they are absolute synonyms or not. Cruse notes that "two lexical units would be absolute synonyms (would have identical meanings) if and only if all their contextual relations were identical" [1]. In this case there could be said that any difference in meaning whatsoever disqualifies a pair of lexical items from being absolutely synonymous. If we think in this way, is it possible for anyone to command the whole lexis of particular language along with all possible meanings of every single word? It is impossible because language is "a living system" which changes.

Absolute synonymy, if it exists, is extremely uncommon and necessarily temporal, e.g. one word obtains another new meaning, or one of its meanings is started to be used in new context. In the case of translation, it would be too good to be true, if absolute synonymy existed across different languages. It is not possible, as languages differ from one another in the same way as words within particular language. Motivation for existence of absolute synonymy is not also obvious; one of the lexical units would be used more frequently and the other would gradually fall into the state of obsolescence. When we think about the sentence structure, it is extremely difficult to find synonymy in words but contextual synonymy is even more unlikely.

3. Conclusion

We are not able to say with 100% certainty if absolute synonymy exists or not. The truth is somewhere in the middle. Only in theoretical way it is possible for us to speak about something that is ABSOLUTE. Synonymy in general is "natural feature" of each language; it is evident that CERTAIN meanings of words overlap. On the other hand absolute synonymy means that two words, two lexical items are substitutable in any contexts constantly; in short, everything what one word means is what the other word means no more and no less. And if we consider this, then we realize that it is not possible. In artificial conditions, where language has a finite number of words, each word has a finite number of meanings, there is a finite number of contexts, such language has to be isolated, closed system without any changes. Then maybe it could be possible to start searching for such words. But we cannot imagine who would be able to examine it, word by word, meaning by meaning, context by context. Anyway, it is a utopia. Every single word in language is unique and has its own identity. However, it is not impossible that someday somehow somebody might find such words (absolute synonyms) and nobody would have a different opinion. It might be the exception that proves the rule.

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Partial Covering of a Sphere with Random Number of Spherical Caps

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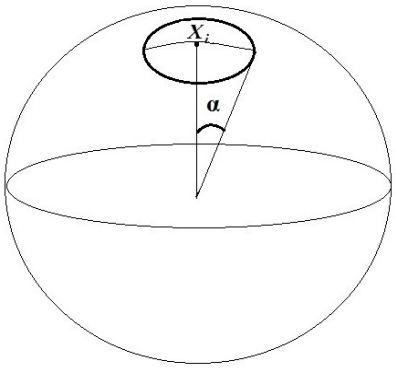
Abstract. In this paper we study sphere coverage problem. A sphere of radius one in a 3-dimensional Euclidean space is given. We consider random location of *N* spherical caps on a sphere, under assumption that *N* is a discrete stochastic variable with a Poisson distribution. The expected value of the length of boundary of covered area is investigated. The authors consider the sequential covering of identical spherical caps, such that none of them contains the center of another one. Sphere covering is considered according to the number of spherical caps.

Keywords: Random covering, spherical caps, Poisson distribution, expected value.

1. Introduction

The goal of this work is to obtain the probability characteristics of the random covering of the sphere by spherical caps. Let S^2 be a sphere of radius one in $\mathbb{R}^3$ and let points $X_i, i \in \{1,2,\ldots,n\}$ be randomly located on S^2. A spherical cap is a portion of a sphere cut off by the plane. By $D(X_i;\alpha)$ we mean a spherical cap of the angular radius $\alpha \in (0,\pi)$ with center at the point X_i without a boundary (see the below picture).



The probability space $(M(S^2),B(M),P)$ is an appropriate model of the random location of a number of spherical caps on a sphere. Here, $M(S^2)$ denotes the space of Lebesgue measurable subsets on $S^2 \subset \mathbb{R}^3$ with finite Lebesgue measure μ of its subsets. In our investigation we assume that the Lebesgue measure of the spherical cap is the length of its boundary. An outcome is the result of a single execution of the model. Since individual outcomes might be of a little practical use, more complex events are used to characterize groups of outcomes. The collection of all such events forms σ-algebra. We consider Borel algebra $B(M)$ of the class of all Lebesgue measurable sets on $M(S^2)$. This can be done with the use of the probability measure function P. Here P is the assignment of geometrical probabilities to the events A, that is, a function from events to the probability levels defined as follows

$$P(A)=\frac{\mu(A)}{\mu(S^2)}.$$

In Poisson point processes the probability, where random value $\xi:M(S^2)\to\mathbb{R}$ equals i, is given by the following formula

$$P(\xi(A)=i)=\frac{(\lambda\mu(A))^i e^{-i\lambda\mu(A)}}{i!},$$

where $\mu(A)$ length of the boundary A. The It is well known that the positive real number $\lambda\mu(A)$ equals the expected value $E(\xi(A))$. So, we have

$$E(\xi(A))=\lambda\mu(A).$$

The following law of total probability is used in our investigations.

Lemma 1. *The event A in the probability space is given. Hence*

$$P(A)=P(A\mid\xi=i)P(\xi=i),$$

where $P(A|\xi=i)$ is conditional probability and $P(\xi=i)$ is probability by the event $(\xi=i)$.

For discrete random variable ξ the expected value $E(\xi(A))$ can be found in the following formula:

$$E(\xi(A))=\sum_{i=0}^{\infty}P(\xi(A)\mid\xi=i)P(\xi=i). \qquad (1)$$

Let $X_i \neq X_j$ where $i,j\in\{1,2,\ldots,n\}$ and $D(X_i;\ \alpha)\cap D(X_j;\ \alpha)\neq\emptyset$. The point which belongs to the boundaries of more than one spherical cap is called the boundaries intersection point. Any of two spherical caps have either two boundaries intersection points or none. Boundaries intersection point is not covered by any of these caps. Such point may be covered only by another spherical cap. Sphere S^2 is covered by spherical caps if each boundaries intersection point is covered by a spherical cap.

In our investigation, by the random variable $\xi:M(S^2)\to\mathbb{R}$ of event A we mean a number of boundaries intersection points in the fixed region $(\xi(A)<\infty)$.

Theorem 1. *The expected length of the boundary of area covered by spherical caps $D(X_i;\alpha)$, $i\in\{1,2,\ldots,N\}$ randomly located in S^2, where N is fixed and equal to n and α is a constant, is given by the following equation:*

$$E_L(n+1)=\frac{1}{n}E_L(n)(n-1)\left(\cos\frac{\alpha}{2}\right)^2.$$

Proof. Let us consider N spherical caps $D(X_i;\alpha)$, $i\in\{1,2,\ldots,N\}$ randomly located on S^2, where N is random variable with Poisson distribution. The expected value of the number of spherical caps equals the parameter of the Poisson distribution $\lambda\mu(S^2)$, i.e. $\lambda 4\pi$. The length of boundary of one spherical cap is $2\pi\sin\alpha$. Thus, expected value of the length of boundary of the union of spherical caps randomly located on S^2 equals $8\pi^2\sin\alpha$.

Since that is shown in [3], probability of the event that any point on S^2 is not covered equals $e^{-4\pi\sin^2\frac{\alpha}{2}\lambda}$ (see Lemma 4 in [3]), then expected length of the boundary is

$$E_L(\lambda)=8\lambda\pi^2\sin\alpha e^{-4\pi\lambda\sin^2\frac{\alpha}{2}}. \qquad (2)$$

On the other hand, from (1) we have

$$E_L(\lambda)=\sum_{n=0}^{\infty}E_L(N=n)\frac{(4\pi\lambda)^n e^{-4\pi\lambda}}{n!}. \qquad (3)$$

By comparing (2) and (3) and multiplying both sides by $e^{4\pi\lambda}$ we obtain

$$8\lambda\pi^2 \sin\alpha e^{-4\pi\lambda\sin^2\frac{\alpha}{2}} e^{4\pi\lambda} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n e^{-4\pi\lambda}}{n!} e^{4\pi\lambda}.$$

It implies, that

$$8\lambda\pi^2 \sin\alpha e^{4\pi\lambda\left(1-\sin^2\frac{\alpha}{2}\right)} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!}.$$

Hence we have

$$8\lambda\pi^2 \sin\alpha \sum_{n=0}^{\infty} \frac{(4\pi\lambda)^n\left(1-\sin^2\frac{\alpha}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!},$$

$$2\pi \sin\alpha \sum_{n=0}^{\infty} \frac{(4\pi\lambda)^{n+1}\left(1-\sin^2\frac{\alpha}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!},$$

$$2\pi \sin\alpha \sum_{n=1}^{\infty} \frac{n(4\pi\lambda)^n\left(1-\sin^2\frac{\alpha}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!},$$

$$2\pi \sin\alpha \sum_{n=0}^{\infty} \frac{n(4\pi\lambda)^n\left(1-\sin^2\frac{\alpha}{2}\right)^{n-1}}{n!} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!},$$

$$\sum_{n=0}^{\infty} \frac{(4\pi\lambda)^n 2\pi\sin\alpha n\left(1-\sin^2\frac{\alpha}{2}\right)^{n-1}}{n!} = \sum_{n=0}^{\infty} E_L(N=n)\frac{(4\pi\lambda)^n}{n!}.$$

From the above

$$E_L(N=n) = 2\pi\sin\alpha n\left(1-\sin^2\frac{\alpha}{2}\right)^{n-1}.$$

Finally, we get

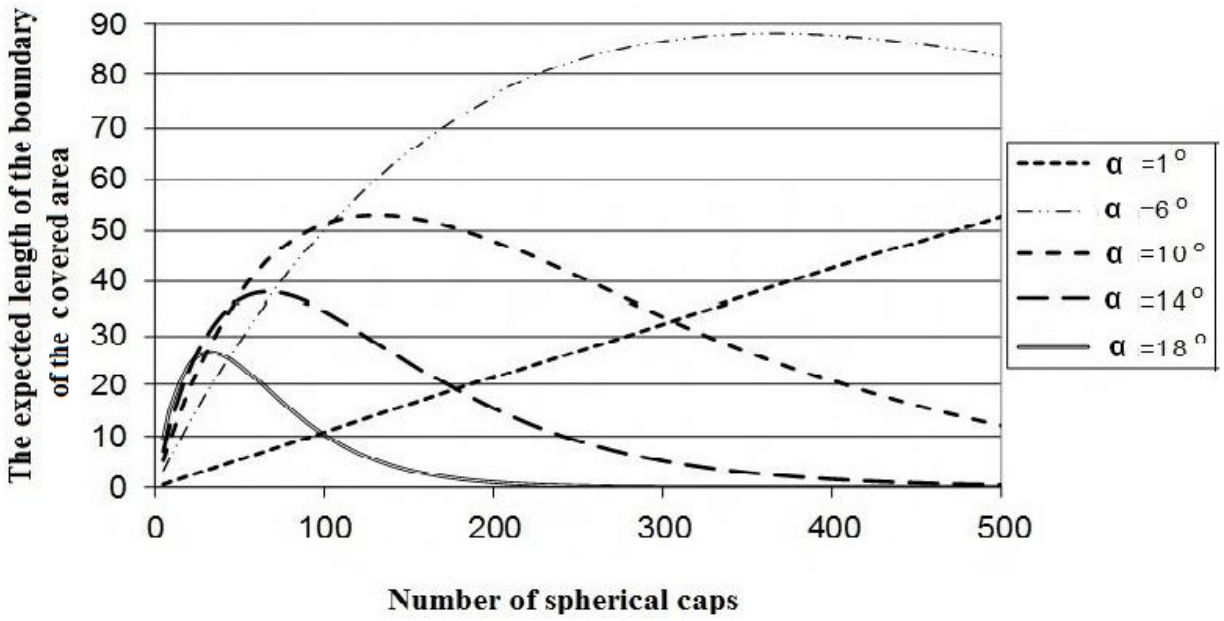
$$E_L(N=n) = 2\pi n\sin\alpha\left(\cos^2\frac{\alpha}{2}\right)^{2n-2}.$$

Replacing $E_L(N=n+1)=E_L(n+1)$ we obtain

$$E_L(n+1) = \frac{1}{n}E_L(n)(n-1)\left(\cos\frac{\alpha}{2}\right)^2.$$

Remark 1. $\lim_{n\to\infty} E_L(n) = 0.$

It means that since a large number of spherical caps always fully cover the sphere then the covered area has no boundary. In such case the expected value of the length of boundary is zero. This property is shown in the following graph, too which shows the expected length of the boundary of area of a sphere which is a sum of n spherical caps on S^2.



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Orthogonal Polynomials Useful in Engineering Computations

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Abstract. In the contribution we will point out the importance of some classes of orthogonal polynomials in the problems of engineering computations, e. g. in civil engineering computations and in electrical engineering computations. We concentrate our attention in the contribution to classical Jacobi orthogonal polynomials, classical Legendre polynomials and generalized brothers of them.

Keywords: orthogonal polynomial, Jacobi polynomial, Legendre polynomial, generalized Legendre polynomial.

1. Introduction

Theory of orthogonal polynomials is the important part of mathematics used especially in the approximation theory, probability theory and statistics. Also new parts of mathematics, as wavelets and fractional analysis are, deal with special functions involving some classes of orthogonal polynomials.

Now we recall the definition and basic properties of orthogonal polynomials (cf. [10]).

Definition 1. Let $(a,b)\subset R$ be a finite or infinite interval. A function $w(x)$ is called the weight function if at this interval it fulfills the following conditions:

(i) $w(x)$ is nonnegative at (a,b), i.e.

$$w(x)\geq 0,$$

(ii) $w(x)$ is integrable at (a,b), i.e.

$$0<\int_a^b w(x)\,dx<\infty$$

and

(iii) for every $n=0,1,2,\ldots$

$$0<\int_a^b |x|^n w(x)\,dx<\infty .$$

Definition 2. Let $\{P_n(x)\}_{n=0}^{\infty}$ be a system of polynomials, where every polynomial $P_n(x)$ has the degree n. If for all polynomials of this system

$$\int_a^b P_n(x)P_m(x)w(x)dx=0,\quad n\neq m,$$

then the polynomials $\{P_n(x)\}_{n=0}^{\infty}$ are called orthogonal in (a,b) with respect to the weight function $w(x)$. If moreover for every $n=0,1,2,...$

$$\|P_n(x)\|_{w(x)}=\left[\int_a^b P_n^2(x)\,w(x)dx\right]^{\frac{1}{2}}=1,$$

then the polynomials are called orthonormal in (a,b) with respect to $w(x)$.

Remark 1. The condition of the orthonormality of the system $\{P_n(x)\}_{n=0}^{\infty}$ has the form

$$\int_a^b P_n(x)P_m(x)w(x)dx=\delta_{nm},$$

where δ_{nm} is the Kronecker delta.

2. Weight Functions of Orthogonality for Classical Jacobi Polynomials, Classical Legendre Polynomials and Generalized Legendre Polynomials

It is well-known that classical Jacobi polynomials $\{P_n(x;\alpha,\beta)\}_{n=0}^{\infty}$ are orthogonal in the interval $I=(-1,1)$ with respect to the weight function

$$J(x)=(1-x)^{\alpha}(1+x)^{\beta},\ x\in(-1,1) \tag{1}$$

where $\alpha>-1$, $\beta>-1$. Very important special case of Jacobi polynomials are classical Legendre polynomials $\{P_n(x;0,0)\}_{n=0}^{\infty}$, for which $\alpha=\beta=0$ in the weight function $J(x)$. In the next we denote them by $\{P_n(x)\}_{n=0}^{\infty}$. As it is seen, they are orthogonal in $I=(-1,1)$ with respect to the weight function $L(x)=1$.

In [6] we introduced the system of polynomials $\{Q_n(x)\}_{n=0}^{\infty}$ orthonormal in I with respect to the weight function

$$Q(x)=\left(x^2\right)^{\gamma},$$

where $\gamma>0$ and $Q_n(+\infty)>0$. It is clear that these polynomials are generalization of the classical Legendre polynomials, which can be obtained by substituting $\gamma=0$ in the weight function $Q(x)$.

Further in [6], we introduced two classes of orthonormal polynomials:

(a) polynomials $\left\{P_n\left(x;0,\gamma-\frac{1}{2}\right)\right\}_{n=0}^{\infty}$ orthonormal in I with respect to the weight function

$$J_1(x)=(1+x)^{\gamma-\frac{1}{2}} \tag{2}$$

and

(b) polynomials $\{P_n(x;0,\gamma)\}_{n=0}^{\infty}$ orthonormal in I with respect to the weight function

$$J_2(x)=(1+x)^{\gamma} \tag{3}$$

In both these cases we have classical Jacobi polynomials orthogonal with the weight function (1) for $\alpha=0$, $\beta=\gamma-\frac{1}{2}$ and $\alpha=0$, $\beta=\gamma$, respectively. In [6] we established relations between the above systems of polynomials expressed in the following theorem.

Theorem 1. For the polynomials $\{Q_n(x)\}_{n=0}^{\infty}$ and special cases of classical Jacobi polynomials orthogonal with the weight functions (2) and (3) we have

$$Q_{2n}(x) = 2^{\frac{\gamma}{2}-\frac{1}{4}} P_n\left(2x^2-1; 0, \gamma-\frac{1}{2}\right) \tag{4}$$

and

$$Q_{2n+1}(x) = 2^{\frac{\gamma}{2}} x P_n\left(2x^2-1; 0, \gamma\right) \tag{5}$$

3. Some Properties of Introduced Generalized Legendre Polynomials

For our further investigation we will use the following theorem (cf. [10]).

Theorem 2. If the interval of orthogonality is symmetric according to the origin of coordinate system and a weight function $w(x)$ is even function, then every orthogonal polynomial $P_n(x)$ is even and odd, respectively according to the oddness and the eveness of its degree. There holds

$$P_n(-x) = (-1)^n P_n(x).$$

Proof. E. g. in [10].

Theorem 3. The polynomials $\{Q_n(x)\}_{n=0}^{\infty}$ satisfy the following relations:

(i) $Q_n(-x) = (-1)^n Q_n(x)$,

(ii) $Q_{2n}(-x) = 2^{\frac{\gamma}{2}-\frac{1}{4}} P_n\left(2x^2-1; \gamma-\frac{1}{2}, 0\right)$,

(iii) $Q_{2n+1}(-x) = -2^{\frac{\gamma}{2}} x P_n\left(2x^2-1; \gamma; 0\right)$,

(iv) $Q_{2n}(1) = 2^{\frac{\gamma}{2}-\frac{1}{4}}\binom{n+1}{n}$ and $Q_{2n+1}(1) = 2^{\frac{\gamma}{2}}\binom{n+1}{n}$

and if k is a natural number, then

(v) $$\frac{d^k}{dx^k} Q_{2n}(x) = 2^{\frac{\gamma}{2}-k+\frac{7}{4}} \frac{\Gamma\left(k+n+\gamma+\frac{1}{2}\right)}{\Gamma\left(n+\gamma+\frac{1}{2}\right)} x P_{n-k}\left(2x^2-1; k; k+\gamma-\frac{1}{2}\right),$$

(vi) $$\frac{d^k}{dx^k} Q_{2n+1}(x) = 2^{\frac{\gamma}{2}-k+2} \frac{\Gamma(k+n+\gamma+1)}{\Gamma(n+\gamma+1)} x^2 P_{n-k}\left(2x^2-1; k; k+\gamma\right),$$

where $\Gamma(x)$ is gamma function.

Proof. Based on the Theorem 1, the Theorem 2, the Theorem 3, the relations (1) – (5) and properties of the classical Jacobi and the classical Legendre polynomials given e. g. in [1].

4. Conclusion

In applications of mathematics involving either the Laplace or the Helmholtz equation in spherical coordinates the associated Legendre equation occurs. Its solutions are called associated Legendre functions. They have some relations to classical Legendre orthogonal polynomials and classical Jacobi orthogonal polynomials. The classical Legendre polynomials and the associated Legendre functions have been used by some authors at the approximation of length of curves and cylindrical surfaces and the approximation of Earth shape in geosciences and engineering computations. Other applications in engineering sciences can be found in [2], [4], [5], [7], [8], [11] and [12].

In this paper we presented several relations which may be used as expressions of the classical Jacobi polynomials of the argument $2x^2 - 1$ by means of generalized Legendre polynomials of the argument x . Other relations for the orthogonal polynomials in general in the forms of recurrence relations can be found in [3] and specially for Jacobi polynomials in [9].

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Transitivity in Political Discourse

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Abstract. This paper deals with the patterns of transitivity in selected examples from the political discourse of George W. Bush. The theoretical part of the paper elaborates the area of Systemic – Functional Linguistics: the system of transitivity. The analytical part discusses the portrayal of individual political actors in discourse. The aim of the article is to observe the creation of the nature of the category of enemy, through considering the types of Material processes that the enemy is represented as taking part in. The focus is also placed on the world-view and actions that the USA represents and assumes.

Keywords: Systemic-Functional Linguistics, transitivity, political discourse, ideational metafunction.

1. Introduction

The 9/11 attacks on the United States became a pretext for a series of overseas military operations executed by the administration of George W. Bush. The main purpose of the military operations was to fight a war against terrorism. As a result of the events of 9/11, there developed a consistent rhetoric aimed at justifying military actions overseas.

This paper explores one of the aspects of this rhetoric, which is associated with the creation of "otherness" and the polarization into two camps: "Us" and "Them" in discourse.

The analytical approach that is adopted in revealing of this polarization is related to the system of transitivity of Systemic – Functional Linguistics developed by M.A.K. Halliday.

The system of transitivity within Systemic-Functional Linguistics (SFG) is concerned with how reality (including fictional and mental reality) is presented in language. This is an area of grammar of a clause which is associated with the conceptualization and depiction of the world around us.

The focus of the paper is on one particular type of process – Material process, and its employment in the discursive structure of two speeches delivered by G. Bush in October 2002.

The speeches that are analyzed may be generally labeled as "call to arms" – within this type of political speech, the leader intends to "inspire patriotic feelings that will overcome other types of reactions such as fear, caution, cynicism, and disbelief." [1, p. 9]

2. Defining Transitivity

In general, transitivity is concerned with meaning and with how meaning is presented in a clause.

The system of transitivity is the element of the lexicogrammar which realises the ideational metafunction (the other two metafunctions of language are interpersonal and textual).

Halliday [2, p. 53] argues that the ideational function of a clause deals with "the transmission of ideas"; and that "the role of the ideational function is to introduce the schemes of 'experiences' or, in the broadest sense, 'processes', which typically include "actions or events of consciousness and relations."

Its main elements are the processes in texts and the types of participants and circumstances that are usually identified with them. As such it is associated with the register dimension of field.

Halliday [3, p. 39] suggests that this function of language which encodes our experience in the form of an ideational content not only sets out the available choices in meaning but also specifies the nature of their structural realizations.

According to Halliday [2, p. 101]

> [a] fundamental property of language is that it enables human beings to build a mental picture of reality, to make sense of their experience of what goes on around them and inside them. Here again the clause is the most significant grammatical unit […] because it is the clause that functions as the representation of processes. […] Our most powerful conception of reality is that it consists of "goings-on": of doing, happening, feeling, being. These goings-on are sorted out in the semantic system of the language and expressed through the grammar of the clause.

According to Halliday [2, p. 106], the transitivity system "construes the world of experience into a manageable set of PROCESS TYPES." Thus, it can be stated that the system is concerned with how reality is presented in language.

Halliday [3, p. 134] defines transitivity as "the set of options whereby the speaker encodes his experience of the processes of the external world, and of the internal world of his own consciousness, together with the participants in these processes and their attendant circumstances; [...] Transitivity is really the cornerstone of the semantic organization of experience."

Transitivity, according to Halliday [in 4, p. 159] includes the whole clause, and it "refers to the "content" or factual – notional structure of the clause in its entirety."

2.1. Transitivity – Concepts and Terms

It is important to clarify particular concepts and terms associated with the system of transitivity. Although the system is termed "transitivity", this area deals with more elaborate elements than just simple difference between transitive and intransitive verbs. Halliday´s model is highly semanticized and it investigates deeper elements of the connection between verbs and the phrases associated with them.

The system of transitivity is not primarily concerned with verbs as being a syntactic category, but on the types of processes represented in the clause, and on how meaning is present, realized and expressed in the clause.

Halliday [in 4, p. 159] argues that the term process is "understood in a very broad sense, to cover all phenomena to which a specification of time may be attached – in English, anything that can be expressed by a verb: event, whether physical or not, state, or relation."

The complete representation of processes, on the basis of this definition, includes the relationships between a particular process and the participants included in it. The participant should be seen as linguistic representations of abstract, or inanimate objects, and human beings as well.

2.2. Types of Processes

Within semantic properties of a text, it is noticeable what types of predicates are present: these are expressions referring to actions, states or processes. Predicates (and participants connected with them) perform the primary responsibility for the presentation of events and actions to which the text refers. According to Bloor and Bloor [5, p. 109] "the process centers on that part of the clause that is realized by the verbal group, but it can also be regarded as what "goings-on" are represented in the whole clause."

Simpson [6, p. 88] notes that "processes can be classified according to their main features: they represent actions, speech, states of mind or simply states of being."

Halliday [2, p.101] argues that "transitivity specifies the different types of processes that are recognized in the language, and the structures by which they are expressed."

The main components of the transitivity system are Material, Mental and Relational Processes.These are the three main types of processes in the English transitivity system. In addition, there are, according to Thompson [7, p. 96], "three less central types which can be distinguished on

the basis of the usual combination of semantic and grammatical criteria.“ These processes are Behavioural, Verbal, and Existential Processes.

For the objective of the analysis, processes are essential, as they deal with forms of verbs and with the character of particular “doings.” The processes as such illustrate important aspects about the manner in which actors and the world are presented in political discourse.

Halliday [2, p. 101] suggests that “the basic semantic framework for the representation of processes is very simple. A process consists potentially of three components:

1. the process itself;
2. participants in the process;
3. circumstances associated with process.” (ibid.)

Halliday [2, p. 102] goes on to argue that the “concepts of process, participant and circumstance are semantic categories which explain in the most general way how phenomena of the real world are represented in linguistic structures.” For the purposes of the analysis, we will focus only on the Material processes.

2.3. Material Processes

This type of process is characterized by the involvement of physical action. A typical interpretation of a verb is “doing word.” According to Butt et al. [8, p. 52], Material processes ”answer the question ”What did X do?” or ”What happened?”

The “doer” of this category of activity is referred to as “Actor.” The Actor does the deed or performs the action. The second participant is labeled as the ”Goal”, and this is because the action, or activity is directed at this entity. Actor and Goal are the two regular participants in clauses associated with the Material processes. The aim of the analysis of Material processes is to expose who is presented as the most powerful entity in discourse.

2.4. Transitivity and Ideology

The conceptual features of ideology are not mirrored only in the vocabulary of a particular language. They function in the grammar of language and they constitute greater “danger“, as they are more covert. The degree to which language governs, rather than depicts experience is one of the essential questions in linguistics.

The system of transitivity also offers a useful device for analysing how political actors render their ideology and how this is transformed and presented in political discourse.

Fowler [9, p. 27] summarizes this feature of meaning in the following:

Transitivity allows to analyse how, by selecting and preferring particular verbal processes to others the author of the text is able to highlight specific meanings in discourse and to restrain other meanings. Linguistic codes do not reflect reality neutrally; they interpret, organize, and classify the subjects of discourse. They embody theories of how the world is arranged: world-views or ideologies.

Thus, the system of transitivity offers vital tools for the examination of how the linguistic organization of a text presents certain views on the world.

These tools are associated with the author of a text and with how a receiver’s understanding of discourse can be directed to a certain direction by the author of a text. In this respect, Fowler [10, p. 71] suggests that “[s]ince transitivity makes options available, we are always suppressing some possibilities, so the choice we make – better, the choice made by the discourse – indicates our point of view, is ideologically significant.“

Transitivity is widely used as an analytic tool within Critical Linguistics– e.g., in relation to the analysis of news, Fowler [10, pp. 231 – 232] states that it “should pay particular attention to how what people say is transformed. There are clearly conventions for rendering speech newsworthy, for bestowing significance on it. Such convention are little understood at the moment.“

3. Analysis of Transitivity in Political Discourse

3.1. Analytical Tools

Within the analysis itself, the focus will be placed on the Material processes with the help of which the nature of the enemy was created and presented in political speeches delivered by George W. Bush.

For this purpose, the concepts of Actor, Process, and Goal from the theory of transitivity of Systemic - Functional Linguistics are used as analytical tools.

I will examine the data from political speeches by George W. Bush with the aim to show how these analytic tools can be used in this type of text analysis. Finally, there will be a discussion about the portrayal of individual political actors in discourse.

Actor	**Process: Material**	**Goal**
The Iraqi regime	has sponsored and sheltered	terrorists;
The regime	has developed	weapons of mass death;
It	has stockpiled	biological and chemical weapons
It	is rebuilding	facilities
The Iraqi regime	possesses and produces	chemical and biological weapons
It	is seeking	nuclear weapons

Tab. 1. Iraq as "Actor" in Material Processes

Actor	**Process: Material**	**Goal**
We	will help rebuild	a liberated Iraq
We	will lead	a coalition
We	will secure	the peace

Actor	**Process: Material**	**Goal**	**Beneficiary**
We	will give	strength	to others
We	will give	hope	to others

Tab. 2. The USA as "Actor" in Material processes

3.2. Discussion

In order to communicate the ideology of war in an effective way, it is vital to present arguments that could be applied discursively in a manner that is easily intelligible and which could gain the widest possible approval by receivers of the texts. One discursive strategy to ensure this goal is to create strong polarization.

The analysis of material, as well as mental processes with respect to participants can show how people and entities are portrayed in discourse. Saddam Hussein and his regime are presented as agents of wide array of violent actions, while the USA is involved in active actions that indicate a sense of duty and mission.

The Goals of the material processes associated with the USA are mostly positive ones, having beneficiary results, such as giving strength or hope. The USA is also projected as the leader of the world, and of the liberating army. This indicates the element of international duty and a challenge to act in a responsible way while making use of readiness and resourcefulness of the USA. At the

same time, there is also a sense of duty due to high obligation. This is supported by the use of the future tense - the United States is involved mostly in actions and processes directed to future.

As it may be expected, the enemy is involved mainly in the material processes of a negative and violent type as if these were naturally occurring actions and phenomena. The Goals of the Material processes associated with enemy are focused on nominal groups that are negatively evaluated (terrorists, nuclear weapons, chemical weapons, chemical attacks, weapons of mass death).

The enemy does not participate in any other processes, such as saying, or sensing, i.e. it is not given any "voice" and it is not presented via any "human actions."

4. Conclusion

The analysis of the Material processes with respect to participants can show how people and entities are portrayed in political discourse. The selections of processes in the system of transitivity in the Bush's speeches play an essential role in the creation of the negative image of the enemy. On the other hand, material processes attribute an active, dynamic role to the USA and indicate a sense of duty and mission - and this fact reflects the purpose of the speeches.

In can be concluded that the choice of verbal types with respect to Actors, Processes and Goals involved in discourse gives the discourse clear ideological structure. This fact reflects the polarization into two opposites groups, and it is achieved through the scope of possibilities within which the participants in discourse are named by means of grammar.

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Professional Community, Language Culture and Standards

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Abstract. The special communication forms a fundamental part of communication activity. The professional community perceives practical and spiritual value of language. The professionals have positive relation to language and try to cultivate it by using it consciously. From the linguistic point of view it is necessary to respect „professional variant of standard". The codifiers have to work promptly, flexibly, without unnecessary delays. They have to bear well-founded suggestions from the professionals.

Keywords: Professional community, language standards, practical and spiritual value of language, professional variant of standard, codifier, cultivation of special language.

1. Introduction

The common and special communication have become the center of attention not only of scientists but of real users, too. The development of society has also been reflected in language, substantial changes have occurred in the language competence and the relations of written and oral communication which are realised by standard language have got closer. The language condition, meaningful and systematic interest in language in practice, language education and respect in the society are the preconditions for improving language culture, dynamic development of language and strengthening its national and representative function.

The special communication forms the substantial part of work of everyday communication specialists. A lot of terms have lost their specificity, they have become the normal part of the common communication. The active and effective terminological and determinological processes are ongoing, the terms and words are migrating from one to another terminological area, they are losing their original meaning and taking a new one. As a consequence of a considerable scientific and technical development, the special communication has increased its importance. At present it forms more than 80% of information spread by new mass media.

The cooperation linguistics, psychology and sociology (social studies) formed the essentials for the identification of a specific language user. Who is he/she? What society is he/she part of? What value does he/she see in language? Is a good command of a native and foreign language a matter of prestige for him/her? These are some of the questions we ask and look for answers to them if we want to find his/her relation to the language standard.

„We can define the language competence or natural mastery of language system and language for specific purposes as an ability to describe the units and structures of language, to produce new units and structures and to interpret the understood units and structures" (Dolník, 2003 a, s. 3). The ability to describe, produce, interpret and simulate is undoubtedly connected with the technical education and with innate intelligence.

The specialists communicate about the activities, processes, innovative problems and the need of new designations arisen in the concrete situations. The common and special language are under massive pressure of internationalization. The amount of internationalisms entering the language from English raises a lot of doubts about their usefulness. Mainly the linguists are afraid of the loss of „language purity" and emphasize the redundancy of the new means of expression or doublets. It might be advisable to rely on the language intuition of the professional community. Its members can

find a new value in the means of expression, which can detail the term or extend the spectrum of usage.

The users of the special language present the results of their activities before professional community (workshops, lectures, conferences etc.) or in popular form before general public in mass media. The written texts are well-considered, prepared with rational aim. They have passed special and language review. A special attention is paid to oral presentation, too. If the authors want to reach optimal goal they prepare the contents, choose a convenient strategy and adequate means of expression. The high-quality lexicon, comprehensible for the audience is an inevitable condition for most of them.

The language has *a **practical value*** for the professional community. The needs of users include the needs to experience life, to widen the knowledge, to identify with a particular social group, to enlarge and popularize their learning and skills through the language. Their motivation to use standard language is connected with its ***spiritual value.*** The professionals are not closely concentrated and ruthless pragmatists. They form an entity which is a part of social group and has a common culture, territory, history, and they are identified and socialized within the bounds of a community. The awareness of a spiritual value of a language, mainly of a native language is an issue that primarily academic community deals with. Nowadays the language users can present the results of their activities abroad, can participate in postgraduate studies at foreign universities and confront themselves with foreign culture and mentality. In such situations they understand the spiritual dimension of language more intensively.

Do the users of special language participate in its cultivation? The condition of the development of sciences and technology asks questions connected with the ***intellectualization of the standard language.*** It is the level of language development when language speech becomes more accurate and it is able to express whole complexity of minds and their connection. In other words, it expresses strengthening of an intelectual aspect of language with the ***language of science*** being on the highest level of it. The language of science and technology respects three basic linguistic principles: ***systematic nature, prescriptivness, aesthetic requirements.*** These are internal language principles which can be supplemented with general social principles and the accepted codification which is based on the language principles as a theoretical base.

The peculiarity of the development of terminology – its dependence on the development of the concrete science significantly influences the ***„social life“*** of terms, creates problems in the standardization of terminology in the theoretical and practical aspects. This raises questions about the place of terminology in the structure of the contemporary language, about the fundamentals of terms as signs of special concepts, about the basic features of terms, tendencies of lexical-semantic development and lexis formation for specific purposes and phraseology.

The terminology is given a right of some independence in all its basic linguistic characteristics and tendencies of development. The independence results from the autonomy of the system as an integral unit with terminology inputs as a fundamental part.

If the terms are given the right of independence in their formation and development, there is a certain independence of linguistic criteria in their evaluation. In the field of term formation there are common tendencies leading to the maximum of specialization, the extension of regularity of word-formative morphemes and models for the expression of certain denotation and then creating one´s own terminological word-formative lexicon. In the evaluation of correctness of formation of concrete terms or a whole terminological system it is necessary to overcome the common tendencies.

The language of science only uses the standard language in a way that is suitable for the practical needs of communication for specific purposes, first of all for its terminological vocabulary. If the word-formative formants are important for the semantic specialization of a term, they are used actively. In the case of need of using the formant which is not in the forming system of the existing language it can be taken from other sources (international fond) or it is formed artificially. This can not be qualified as an abnormality of standard or as a disharmony with

standard. It is becoming a standard of the language of science which we can name ***„professional variant of standard".*** The professional variant of standard combines the language of science as the peculiarity of the standard language with the structures of a standard language necessary for all its functional variants.

The professional variant of standard preserves and reflects previous historical conditions of loanwords, traditions, development of their usage in science and technology. A lot of them recede from the field of special communication. It is a naturally irreversible process which is connected with changing of generations and it is under the influence of general language standards. All the processes of contemporary scientific and technical movement are directly reflected in the formation of new terminology and in the complicated transformation of present terms and comprehensive terminological systems. The interactive processes of mutual influence of research methods lead to the synchronic usage of terms within other terminologies.

The contemporary processes of forming and functioning of terminological systems raise expressive demands on the terms, the users of specialized terminology, on the professionals who are concerned with practical questions of term formation. Here we can find the answer to the question about the influence of the professionals on the cultivation of a specialized language. The users of the language participate in the stabilization of the language system, they appreciate the necessity of its orderliness through the usage of a literary language in a correct way i.e.with respect to its codified standards. The problem rises in the codification of terms when a conflict starts in the terminological board between a codifier and a specialist concerning the practical usage of terms. The turbulent movement in the technical and scientific world brings demands on effective means of expression which spontaneously and naturally respond to new situations in all communication fields. For the reasons of communication economy there are often used professionalisms which were recently parts of substandard words. It is impossible to wait for the results of activities of the terminological boards for a long time, to wait whether they approve of the terms, codify the proposed terms and include them to the corresponding standards or change them completely.

2. Conclusion

The standard language has a very strong position. The circumspection (very often inadequate slowness) in the procedure of approval and incorporating new terms into standards and language system can be considered needless. The highly qualified specialists – non-linguists are aware of the importance of language. They are prepared to respect all the aspects which stabilize the standard language. They expect promptness, flexibility and high professionalism from the institutions which have the main say in the codification of terms. The professionals assume that the codification criteria are subject to change depending on the position of the „main codifier". For the experts in the fields of science and technology it is complicated to enforce and establish subject-specific terms which do not describe detailed features, processes, equipment, etc. This situation calls for respect from the theorists-linguists to the professional public i.e. the respect to a large community that uses, cultivates and appreciates the language.

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Unbounded solutions of a linear differential equation with two delays in the critical case

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Abstract. Asymptotic behavior of solutions of a linear homogeneous differential equation with two delayed arguments in the form

$$\dot{y}(t) = \beta(t)\left[y\left(t-\delta\right) - y\left(t-\tau\right)\right]$$

is discussed for t → ∞. It is assumed that β: $[t_0 - \tau, \infty) \to (0, \infty)$, $t_0 \in \mathbb{R}$ and δ, τ ∈ (0, ∞), τ > δ. Known sufficient conditions of the existence of an unbounded solution are improved. The better boundary of the critical value is given for the function β. So far, the obtained results are presented and compared with new results.

Keywords: Unbounded solution, sufficient conditions, critical case, two delayed arguments, discrete delays

1. Introduction

In this paper there is discussed the asymptotic behavior of solutions of a linear homogeneous differential equation with two delayed arguments containing discrete delays

$$\dot{y}(t) = \beta(t)\left[y\left(t-\delta\right) - y\left(t-\tau\right)\right] \qquad (1)$$

as $t \to \infty$. In (1) there is assumed that β: $I_{-1} \to \mathbb{R}^+ := (0, \infty)$ is continuous function, $I_{-1} := [t_0 - \tau, \infty)$, $t_0 \in \mathbb{R}$ and $\delta, \tau \in \mathbb{R}^+$, $\tau > \delta$. The symbol "˙" denotes (at least) the right-hand derivative and $I := [t_0, \infty)$. Similarly, if necessary, the value of a function at a point of I_{-1} is understood (at least) as the value of the corresponding limit from the right. Moreover in this paper there is assumed without loss of generality that $t_0 - \tau > 0$ (this is a technical detail necessary in some of the next computations).

A function y: $[\upsilon - \tau, \infty) \to \mathbb{R}$ will be said a solution of (1) on $[\upsilon - \tau, \infty)$ with $\upsilon \in I$ if $y(t)$ is continuous on $[\upsilon - \tau, \infty)$, continuously differentiable on $[\upsilon, \infty)$ and satisfies (1) for all $t \in [\upsilon, \infty)$.

Let $\mathbb{C}$ be the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}$.

For $\upsilon \in I$ and $\varphi \in \mathbb{C}$, we say that $y(\upsilon, \varphi)$ is a solution of (1) through (υ, φ) if $y(\upsilon, \varphi)$ is a solution of (1) on $[\upsilon - \tau, \infty)$ and $y(\upsilon, \varphi)(\upsilon + \theta) = \varphi(\theta)$ for $\theta \in [-\tau, 0]$.

Asymptotic behavior of solution of (1) has been analyzed in the papers [6, 7]. In the paper [6] there was proved the following condition for the existence of an increasing and unbounded solution of (1) as $t \to \infty$. In this condition there is assumed that there exists a constant $p \in (0, 1)$ such that the inequality

$$\beta(t) \geq \frac{1}{\tau - \delta} - \frac{p}{2t} \qquad (2)$$

holds for all $t \in I_{-1}$. Moreover, in the paper [7] there was proved the convergence of all the solutions of (1) which is characterized by the existence of an increasing and bounded solution of (1) as $t \to \infty$. It was shown that such solution exists if exists a $p > 1$such that the inequality

$$\beta(t) \leq \frac{1}{\tau - \delta} - \frac{p\left(\tau + \delta\right)}{2\left(\tau - \delta\right)t}$$

holds for all $t \in I_{-1}$.

More information about asymptotic behavior of solutions of delayed differential and difference equations and systems can be found, for example, in [1-5], [8-10] and the references therein.

The main result of this paper concerns the existence of an unbounded solution of (1). Especially, there is investigated the so called critical case for the function β which separates the case when all solutions are convergent and the case when there exists an unbounded solution. This investigation is based on the methods and results of the papers [4-8]. These methods and results are presented and applied to improve the sufficient conditions for the existence of an unbounded solution of (1).

The paper is organized as follows. In the Section 2 an auxiliary inequality is given and there are formulated lemmas which are necessary for determining of the sufficient conditions for existence of an unbounded solution of (1). The main result is proved in the Section 3. In the section 4 the conclusion is formulated.

2. Auxiliary Inequality and Lemmas

The auxiliary inequality

$$\dot{\omega}(t) \le \beta(t)\left[\omega(t-\delta)-\omega(t-\tau)\right] \tag{3}$$

plays a major role in the analysis of (1). The properties of solutions of (1) are compared with the properties of solutions of (3).

A function ω: $[\upsilon - \tau, \infty) \to \mathbb{R}$ will be said a solution of (3) on $[\upsilon - \tau, \infty)$ with $\upsilon \in I$ if $\omega(t)$ is continuous on $[\upsilon - \tau, \infty)$, continuously differentiable on $[\upsilon, \infty)$ and satisfies (3) for all $t \in [\upsilon, \infty)$. If

$$\lim_{t\to\infty} \omega(t) = \infty$$

then, the solution ω of (3) is unbounded.

Moreover, there is needed on the analysis of asymptotic behavior of solutions of (1) on the basis of the properties of solutions of (3) to determine a relation between these solutions.
In [6] there is proved the following relation between solutions of (3) and (1).

Lemma 1. Let $\omega(t)$ be a solution of (3) on I_{-1}. Then, there exists a solution $y(t)$ of (1) on I_{-1} such that the inequality

$$\omega(t) < y(t)$$

holds on I_{-1}.

In the following Lemma 2 there is showed that a solution of (3) exists in an exponential form. This result immediately follows from (3) if a solution is looked in the below exponential form (5).

Lemma 2. Suppose that there exists a function ε: $I_{-1} \to \mathbb{R}$, continuous on $I_{-1} \setminus \{t_0\}$ with at most first order discontinuity at the point $t = t_0$ and satisfying on I the inequality

$$\varepsilon(t) + \beta(t)\exp\left[-\int_{t-\tau}^{t} \varepsilon(s)\,ds\right] \le \beta(t)\exp\left[-\int_{t-\delta}^{t} \varepsilon(s)\,ds\right]. \tag{4}$$

Then, there exists a solution $\omega(t) = \omega_e(t)$ of (3), defined on I_{-1}, and having the form

$$\omega_e(t) = \exp\left[\int_{t_0-\tau}^{t} \varepsilon(s)\,ds\right]. \tag{5}$$

In [7] there is derived from (1) and properties $\beta(t) > 0$ for $t \in I_{-1}$, $\tau > \delta > 0$, the following auxiliary statement.

Lemma 3. Let $\varphi \in \mathbb{C}$ be increasing (non-decreasing, decreasing, non-increasing) on $[-\tau, 0]$. Then, the corresponding solution $y(\upsilon, \varphi)(t)$ of (1) with $\upsilon \in I$ is increasing (non-decreasing, decreasing, non-increasing) on $[\upsilon - \tau, \infty)$.

The next auxiliary statement is obtained from Lemma 1 and Lemma 3.

Lemma 4. Let $\omega(t)$ be an increasing and unbounded solution of (3) on I_{-1}. Then, there exists an increasing and unbounded solution $y(t)$ of (1) on I_{-1}.

3. Sufficient Conditions for Existence of an Unbounded Solution of (1)

In this part there are derived from previous results the following easily verifiable sufficient conditions for existence of an unbounded solution of (1).

Theorem 1. Suppose that there exist constants $p \in (0, 1)$ and $\theta > 1$ such that the inequality

$$\beta(t) > \frac{1}{\tau-\delta} - \frac{p(\tau+\delta)}{2(\tau-\delta)t} + \frac{p^2\theta(\tau+\delta)^2}{4(\tau-\delta)t^2} \tag{6}$$

holds for all $t \in I_{-1}$. Then, there exists an increasing and unbounded solution $y(t)$ of (1) as $t \to \infty$.

Proof. It is easy to see that the function of the form (5) is increasing and unbounded on I_{-1} if $\varepsilon(t) = a/t$ for $t \in I_{-1}$, where $a > 0$.

$$\omega_e(t) = \exp\left[\int_{t_0-\tau}^{t} \varepsilon(s)\,ds\right] = \exp\left[\int_{t_0-\tau}^{t} \frac{a}{s}\,ds\right] = \left(\frac{t}{t_0-\tau}\right)^a.$$

Now there is needed to verify when this function is the solution of (3). Lemma 2 is employed. Before, an auxiliary asymptotical analysis is performed with sufficient accuracy for $t \to \infty$.

$$-\int_{t-\gamma}^{t} \varepsilon(s)\,ds = -\int_{t-\gamma}^{t} \frac{a}{s}\,ds = \ln\left(1-\frac{\gamma}{t}\right)^a = \ln\left(1 - a\,\frac{\gamma}{t} + a(a-1)\frac{\gamma^2}{2t^2} + \mathrm{O}\left(\frac{1}{t^3}\right)\right). \tag{7}$$

The symbol "O" used above is the Landau order symbol and γ is nonzero constant. Now integral inequality (4) is rewritten and simplified with using (7) and above ε.

$$\frac{a}{t} \le \beta(t)\left(a\,\frac{\tau-\delta}{t} + a(a-1)\frac{\delta^2-\tau^2}{2t^2} + \mathrm{O}\left(\frac{1}{t^3}\right)\right). \tag{8}$$

Inequality (8) is multiplied by t and a^{-1}, and after some simplifications and substitution $a = 1 - p$, where $p \in (0, 1)$, there is got

$$\beta(t)(\tau-\delta)\left(1 + \frac{p(\tau+\delta)}{2t}\right) > 1. \tag{9}$$

Finally, inequality (9) is modified as follows

$$\beta(t) > \frac{1}{\tau-\delta}\,\frac{1}{1-\left(-\frac{p(\tau+\delta)}{2t}\right)} = \frac{1}{\tau-\delta}\left(1 - \frac{p(\tau+\delta)}{2t} + \frac{p^2(\tau+\delta)^2}{4t^2} + \mathrm{O}\left(\frac{1}{t^3}\right)\right).$$

There is proved that if (6) holds then, there exists an increasing and unbounded solution of (3) which is in the form (5). Consequently, there exists (by Lemma 4) an increasing and unbounded solution of (1).

4. Conclusion

If (6) is compared with (2) then, there is seen that the progress has been achieved. Sufficient condition of the existence of an unbounded solution of (1) has been improved. The better boundary of the critical value has been given for the function β.

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The Rules of the Slovak Ortography in Copywriting

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Abstract. The current form of the national language presented in the electronic media is analyzed primarily in radio and television. Less explored area is the area with copywriting's texts. These marketing texts are not subject to strict requirements of the level of control by language proofreading. The contribution is focused on the part of copywriting - the presentation of the company on Internet website. We analyze the most frequently occurring misspelling in the texts. We focus on the part of the Slovak orthography and its rules of writing. The study provides interesting results and reveals weaknesses in this area.

Keywords: Copywriting, rules of the ortography, misspelling.

1. Introduction

Information and communication technologies have brought new options in the communication. On the Internet a new type of communication was created on the basis of specific texts that deserves attention of researchers in theory, but mainly practical analysis of texts.

In our study, we are dealing with the current situation of copywriting texts on the Internet in the Slovak language.

The definition of copywriting is characterized by a situation that the existing knowledge of this particular field is not organized in Slovakia and it is not easy to find a definition, which is based on universally valid usage.

T. Kučírek [7] defines the borders and deals with copywriting in Slovakia: "Copywriting is the creation of advertising and commercial texts. These may include 'classical' copywriting, when copywriter creates slogans, names, descriptions and PR articles. Its aim is to increase sales of a product or creating a certain image and communication with the public."

Definitions of copywriting agree that it is a creation of advertisement text and in the article we are focused on the texts for the web. There is also SEO copywriting, where texts are not optimized for the potential customer or user, but for Internet search engines.

As regards its characteristics (targeting the highest number of customers and the ever – increasing access on the Internet), copywriting can be classified as a mass medium.

M. Považaj [8] states that "mass media are able to influence public opinion, to guide views, to create moods and modify attitudes of recipient expression. It is natural that significantly influence public language knowledge. This fact is well recognized by the linguists and therefore they constantly pay attention to the language of the mass media."

Research in the field of mass media and electronic media in Slovakia was undertaken mainly by Ján Findra, Matej Považaj, Ján Kačala, Ábel Kráľ, Ľubomír Kralčák and Pavol Dinka.

2. Focus and Methodology of Research

In our study, we focus on copywriting texts of bigger and smaller companies. In particular, we concentrated on the most common type of errors in these texts, which are presented to the Slovak public.

Our aim was to map the current state of copywriter's text written in the Slovak language on the Internet with regard to the Slovak orthography. Companies in a given electronic environment promote and present themselves, their products, services or events – these were copywriting texts.

We divided the texts into two categories. Category A consisted of texts from web sites of smaller companies or organizations operating regionally or nationwide in Slovakia, but focused mainly on the local market.

Category B consisted of texts from websites of bigger companies or organizations that have been transnational in scope and their main business has been focused mainly on the foreign market and our condition was that they should have a website in Slovak.

The company‘s main website (the homepage) was analyzed in all cases and at least two other subpages depending on the amount of written text. If it were not a text-extensive website, five web pages were analyzed. Therefore, the minimum number of pages analyzed consisted in Category A of at least 120 copywriting texts and in Category B of at least 120 copywriting texts.

The results are displayed in the table below. The table introduces the number of mistakes on each language orthographic levels found in both categories analyzed. Then we evaluated the most common types of mistakes in the texts.

Order	**Types of mistakes**	**Further dividing**	**Number of mistakes found – Category A**	**%**	**Number of mistakes found – Category B**	**%**	**Total number of mistakes found**	**%**
	Punctuation	commas	29	8.79%	16	8.70%	45	**8.75%**
		bracket, slash, colon, question mark	26	7.88%	11	5.98%	37	**7.20%**
		dashes, hyphens	30	9.09%	6	3.26%	36	**7.00%**
		dots	18	5.45%	13	7.07%	31	**6.03%**
		the three dots	6	1.82%	1	0.54%	7	**1.36%**
1	Punctuation together		109	33.03%	47	25.54%	156	**30.35%**
2	Typos		20	6.06%	20	10.87%	40	**7.78%**
3	Diacritics	diacritics, accent marks, caron, acute	21	6.36%	16	8.70%	37	**7.20%**
4	Time data	time and date	20	6.06%	12	6.52%	32	**6.23%**
5	Space		20	6.06%	10	5.43%	30	**5.84%**
6	Indents		15	4.55%	10	5.43%	25	**4.86%**
7	Write capital and small letters		9	2.73%	14	7.61%	23	**4.47%**
8	Characters		5	1.52%	3	1.63%	8	**1.56%**
9	I/Y		2	0.61%	5	2.72%	7	**1.36%**
	Total		**330**	100.00%	**184**	100.00%	**514**	**100.00%**

Tab. 1. Mistakes found in the texts in Category A and Category B.

In the table we can see interesting findings. On the website of small companies in the Slovak Republic a higher number of types of mistakes has been found. We have noticed a difference already in the assessment of the mistakes on the websites of the specific companies – when examining the texts of the Category A, specific smaller companies, we found approximately the same number of mistakes on the websites, even if their type was different. In the bigger

transnational companies, we have found several websites that did not contain any mistakes. On the other hand, there were also other websites flooded with mistakes.

We observed the presence of the different types of mistakes in the texts of both categories. Incorrect form of writing through indents more appeared in the Category A. In this category companies more confused dashes with hyphens; same in the punctuation focused on a bracket, a slash, a colon and a question mark was more frequently mistake than in the Category B. Compared to the second category, there was a lower occurrence of confusion writing capital or small initial letter. There were mainly cases, when they wanted by initial capital letter highlight the importance of the word in the text or focus the reader's attention to it. There were less orthographic grammar mistakes. Overall, we recorded a higher number of mistakes at different language levels in this category.

In the Category B, there were a higher number of the words and the sentences without diacritics, which might be caused by automatic translation of the websites in foreign language, or not spent sufficient attention to the translation (because we observed a difference between the company's homepage and subpages). Here we noticed a higher number of typographical errors, i.e. misspelled words. We assume that it was mainly due to inattention of the text's author. However, in this category we found l fewer mistakes at different language levels. We assume that the reason is that not all smaller companies have got an expert who would take care of the company's online presentation, and, therefore, the texts describing the company to the customer contain more mistakes than copywriting texts of bigger companies. They with higher capital should make sure the company presentation. Usually, they have special financial funds for presentation. However, it does not apply to almost one quarter of the websites analyzed, where the transnational scope of the company had the effect of inadequate translation of all subpages into the Slovak language.

In analyzing the copywriting texts on the web we noticed several other deficiencies not included in the table. The deficiencies have been in the interactive features, broken links, too large volumes of data causing pages to get stuck, few formats supported, and so the pages were of no benefit for all users. The texts sometimes have seemed disjointed – have not been uniformly aligned paragraphs – once on the side, once centered. The complexity of the composition of some sentences and formulation has not been always adjusted to the average reader or expected target audience.

3. Conclusion

We cannot be satisfied with the facts determined. More and more people tend to memorize mistakes occurring in great numbers in texts found on websites as a result of increasing and easier accessibility of the Internet. By frequent repetition of incorrectly used language tools are strengthened in the mind of the recipient. What consequences can or will using of inappropriate, non-literary and incorrect means have on the linguistic awareness of Internet's users, we can only speculate.

The analysis also indicates the difference in the types of mistakes, which appear on the websites of smaller companies operating in Slovakia and transnational companies, mainly from abroad. Smaller companies (in terms of the amount of mistakes in the texts) have had websites approximately on the same level. There have been more varied results among the bigger companies. In terms of mistakes in the text, some copywriting texts have been on the excellent level and we found only a few mistakes after careful analysis. Others have contained excessive number of mistakes.

Mistakes are a natural part in creating a text, but their great quantity, however, can be eliminated by the subsequent checks and corrections (by the author of the text or by another person). Author's self-correction is probably not sufficient for Internet copywriting texts, and is neglected by the state and the current legislation.

The advertising message enhances the specificity of copywriting texts. The value of an eye-catching and creative text is often unnecessarily reduced by orthographic mistakes. The correct harmonization of the content with the form is the first step to success.

We believe that it is important also to focus on this issue, because ICT and electronic communication will still grow in power. It is good to have enough knowledge in this area. Our study would like to contribute with some knowledge in this area, because until we know where the problem is, and the next step can be taken to lead to improvement.

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Solving Quadratic Assignment Problem (QAP) Using Local Search with Simulated Annealing Elements

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Abstract. The Quadratic Assignment Problem is one of the hardest among the combinatorial optimization problems with wide practical applications. Since it is a *NP*-hard problem, it is unthinkable to solve it by exact methods. This paper presents an algorithm for solving the QAP based on well-known meta-heuristics, which are combined and improved, to reach the best possible results. There is detailed description of whole algorithm with explanation of impact of individual steps. Outcomes of the algorithm are compared with several other algorithms on some instances of QAPLIB. As it is obvious from summary, proposed algorithm achieves better results than other algorithms.

Keywords: QAP, meta-heuristics, algorithm, local search, solving.

1. Introduction

The quadratic assignment problem (QAP) was first time introduced by [6] in 1957 as a mathematical model for the location of indivisible economical activities and it is one of the combinatorial optimization problems.

Let us assign n facilities to n locations with the cost being proportional to the flow between the facilities multiplied with their distances. The objective is to allocate each facility at a location such that the total cost is minimized. We are given the flow matrix $A = (a_{ij})$ and the distance matrix $B = (b_{kl})$, both with size of $n \times n$, for this objective. The QAP is to find a permutation π of the set $N = \{1,2,\dots,n\}$, which minimizes

$$z(\pi) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{\pi(i)\pi(j)} b_{ij},$$

what can be equally written as $\min_{\pi \in \Pi_n} z(\pi)$, where Π_n is the set of all permutations of N.

As a demonstrational application of the QAP we can consider campus planning model used by [4]. The problem is planning the sites of n buildings on a campus, where b_{kl} is the distance from building k to building l and a_{ij} is the traffic intensity between buildings i and j (amount of people per time unit who travel between buildings i and j). The objective is to minimize the total walking distance among the buildings per given time unit. Each assignment can be mathematically described by a permutation π of the set N, such that $\pi(i) = j$ means the building i is assigned to the site j. Then, the product $a_{\pi(i)\pi(j)} b_{ij}$ represents the walking distance of people travelling between buildings i and j in a given time unit.

The QAP is one of the hardest combinatorial optimization problems. It is shown [10] that QAP is *NP*-hard and even finding an approximate solution with certain deviation from the optimum cannot be performed in polynomial time. Thus all exact methods and algorithms for QAP (providing always optimal solution) are usable just to solve problems of size smaller than $n = 30$ approximately. Most of the exact algorithms for QAP are based on branch and bound method [9].

But many real applications of QAP are sized much greater than $n = 30$. That's why we need to use non-exact methods – heuristics, which allow us to solve QAPs sized greater. There are several classes of heuristics used to solve QAP we can distinguish, e. g. local search (local improvements) methods [9], construction methods [2], simulated annealing methods [12], tabu search methods [11], genetic algorithms [1] etc.

2. The Algorithm

We chose the local search method as an approach for solving QAP, since it's possible to manage the course of solving very well by this method, as it will be shown below. In general, local search starts from some initial state (initial assignment – permutation of N in QAP) and by scanning its neighborhood with consequent local changes tries to improve current assignment. If better assignment is found, it replaces the current assignment and the search continues.

The neighborhood of a state in QAP (of some permutation $\pi \subseteq \Pi_n$) is considered to be the set of permutations which can be obtained by exchanging two to k facilities (two to k positions in π), where $2 \leq k \leq n$. If k positions are exchanged, the search is referred to as k-opt.

Moreover, there are implemented some elements of simulated annealing to presented algorithm (which will be further denoted as *LSSA – Local Search witch Simulated Annealing elements*), namely, acceptance of worse assignments than current assignment with a probability P and gradual decreasing of value k, which both allow us to escape from local minima.

In each i-th iteration the algorithm works with permutation $\pi_i \subseteq \Pi_n$, which represents current assignment of facilities. The algorithm starts from initial permutation π_1 which is generated completely randomly. For the current permutation π_i are randomly generated L vectors $X = (x_1, x_2, \dots, x_n)$, where $x_m \in \{0,1\}$, $m = 1,2,\dots,n$. In the first iteration of the algorithm, count of variables x_m, such that $x_m = 1$, is set to value $(n-k)$, and count of variables x_m, such that $x_m = 0$, is set to value k, i.e. $X_l = \mathbf{shuffle}\left(\underbrace{1,\dots,1}_{n-k},\underbrace{0,\dots,0}_{k}\right)$, where $l = 1,2,\dots,L$ and function *shuffle* shuffles elements of X_l randomly. We set $k = n$ at the beginning of the algorithm. In the next step, the value of function

$$z_l(\pi) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{\pi(i)\pi(j)} b_{ij} x_i^l x_j^l \qquad (1)$$

is computed for each $l = 1,2,\dots,L$. Notice, that only positions of π appertaining to nonzero elements of X_l are added to the total sum. It means that value z_l represents cost of placing just those facilities i on positions j, where values $x_i^l = x_j^l = 1$. Thus, function z_l returns real impact of placing just chosen facilities on positions given by vector X_l. Subsequently, one of the vectors X_l is chosen, such that value of z_l is minimal for all $l = 1,2,\dots,L$. Let's denote chosen X_l as X^*. Positions of nonzero elements in X^* determine positions in current π which will stay fixed during following local search. The neighborhood of current π in the local search is examined by exchanging randomly k elements on positions of zero elements in X^* (there are exactly k zero elements in X^* - k-opt). Let's denote described local search used by the algorithm as "fixed local search". An example of fixed local search used in the algorithm is shown in Fig. 1.

X^*	=	(1,	0,	0,	1,	0)		X^*	=	(1,	0,	0,	1,	0)
π	=	(**4**,	2,	5,	**3**,	1)	$\xrightarrow{k\text{-opt}\,(k=3)}$	π	=	(**4**,	5,	1,	**3**,	2)
$z(\pi)$	=		219					$z(\pi)$	=		205			

Fig. 1. Example of fixed local search by using vector $\mathbf{X^*}$

There are generated M new permutations π_j, $j = 1,2,\ldots,M$ by local search described above, from which the best one (the permutation with minimal value z) is selected (let's denote selected permutation as π^*) as a candidate for new current permutation. If π^* is better than current π_i permutation in i-th iteration (if $z(\pi^*) < z(\pi_i)$), π^* becomes new current permutation in $(i+1)$-th iteration. In the case that π^* is worse than current π_i (if $z(\pi^*) > z(\pi_i)$), it is accepted as new current permutation with probability P, which is computed regarding to following rules (including simulated annealing idea):

1. if in previous $(i-1)$-th iteration generated π^* was better than π_{i-1} and subsequently accepted as π_i, set

$$P = 0{,}1\frac{1}{|z(\pi^*) - z(\pi_i)|}\cdot\frac{k-2}{n}. \tag{2}$$

2. otherwise set

$$P = \left(1 + 0{,}1\frac{1}{|z(\pi^*) - z(\pi_i)|}\right)\cdot\frac{k-2}{n}\cdot P. \tag{3}$$

Notice, that the longer algorithm cannot find better permutation than π_i, the greater is probability of choosing worse permutation than π_i, but simultaneously the worse permutations π^* are examined during local search, the lower is probability of choosing worse permutation than π_i. Described rule allows us to escape from local minima.

Moreover, after every certain count of iterations, value k is decremented. This step causes that the algorithm explores more "thin" neighborhood after each such decrement, since we assume the algorithm converge into the global minima. In addition, the lower is value k, the lower is probability of choosing worse permutation than π_i, what means that the algorithm will not throw away π^* when it is approaching to finish if better π_i isn't found.

The framework of the algorithm is described as follows (pseudo-code):

```
Init.: i = 1, π_i = rand(1..n), k = n, L = n, M = min(n!, 50), counter = 1;
repeat
   for l in 1..L do:
      X_l = shuffle(1, 1, ..., 1, 0, ..., 0)   (n−k ones, k zeros);
      compute z_l(π_i) (according to (1));
   X* = X_l with best value z_l(π_i);
   for j in 1..M do:
      π_j = fixed_local_search(π_i, X*);
   π* = π_j with best value z(π_j);
   if z(π*) < z(π_i) do:
      i = i + 1; π_i = π*;
   else:
      i = i + 1; π_i = π* with probability P (based on (2) and (3));
   counter = counter + 1;
   if counter == 1000 do:
      if k == 1 do:
         return π_i; TERMINATE;
      else:
         k = k − 1; counter = 1;
until the algorithm was not TERMINATED
```

3. Computational Results

The algorithm was tested on several instances with $n \leq 100$ cited in QAPLIB [3]. Computational results of the algorithm LSSA listed in Tab. 1 are compared with results obtained by some other algorithms proposed by their authors. The table shows percentage deviations of compared algorithms over the best known solutions. Results presented in LSSA column are the best ones out of 10 performed runs of the algorithm.

Problem	***n***	**LSSA**	**Ahuja 132[1]**	**Drezner [5]**	**Ji [7]**
kra30a	30	**0**	0	0	0
nug30	30	**0**	0,070	0	0
tho40	40	**0**	0,320	0,01	0,041
wil50	50	**0,004**	0,070	0,002	0,028
sko64	64	**0**	0,220	0	0
sko81	81	**0,009**	0,200	0,014	0,067
sko100a	100	**0,011**	0,21	0,018	0,051
sko100b	100	**0,005**	0,140	0,011	0,039
sko100c	100	**0,006**	0,200	0,003	0,015
sko100d	100	**0,019**	0,170	0,049	0,022

Tab. 1. Computational results.

The table proves that LSSA algorithm is an efficient method for solving the QAP, since in the most cases it is able to find better solution than other algorithms compared with.

4. Conclusion

Because of high difficulty of the QAP, it is very hard to find an optimal solution for large scale quadratic assignment problems in acceptable time. It is impossible to find optimal solution for about $n > 30$ with exact methods. There are developed many meta-heuristic methods for solving QAPs as the consequence and some of them are mentioned above (by cites). Proposed LSSA algorithm, due its great results listed in Tab. 1, can be considered as full-value alternative to any other method for solving QAP. Moreover, it provides greatly better results than many other methods (even which are not mentioned in Tab. 1), by what LSSA moves a possibility of solving the QAP forwards.

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The Development of Investments in Business Model

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Abstract. The article describes Kalecki's business model which consists of the delay differencial equation. The model deals with a behaviour of investors, their investment orders, the production of capital goods and delivery of finished capital goods. This model is completed by existence of positive solutions of the delay differential equation. The solutions are bounded by positive functions. Actually we can find a solution in some concrete cases.

Keywords: Business model; net investments; delay differential equation; positive solution.

1. Introduction

Michał Kalecki (*1899 Łódź - † 1970 Warszawa) was polish economist who has devoted especially to the theory of business cycles and economic growth theories. He was one of first macroeconomists who applied mathematical models and statistics in various economic problems. In this paper the Kalecki's business model is described by equation

$$J'(t)-\frac{m}{\theta}J(t)+\left(\frac{m}{\theta}+n\right)J(t-\theta)=0,\quad t\geq 0, \tag{1}$$

where $J(t)$ is the net investment, θ is the average progress of investment delay and m,n are positive constants which assign an appropriate slope of investment order function. In our previous research we are interested in the study of the existence of solutions of delay differential equations of the form

$$x'(t)+p(t)x(t)+q(t)x(t-\theta)=0,\quad t\geq t_0, \tag{2}$$

which is identical with (1). In the paper [3] we found some conditions for the existence of positive solutions of the equation above, which are bounded by positive functions. This conditions were summarized into the theorem and some corollaries, which we use to find a positive solution of the Kalecki's business model.

2. Kalecki's Business Model

The model concerns a closed economic system without trend. Kalecki considers the short run. This model describes a behaviour of investors, their investment orders, the production of capital goods and delivery of finished capital goods.

Regarding investment, Kalecki assumes that there is an average progress of investment delay which is equal to a positive constant θ. The progress of investment delay is the time interval between the decision to invest and the delivery of the finished capital goods. Three stages of each investment can be summarized here:

1. investment orders (called I), i.e. all the orders for production of capitall good, both for replacement purpose of capital stock of investors and for net addition to the capital stock;
2. production of capital goods (called A), i.e. gross capital accumulation of investors;
3. deliveries of finished capital goods (called L).

Let *W(t)* be the total amount of unfilled investment orders in time *t*. Each order requires a period of time θ to be filled. *W(t)* equals the sum of all orders made in interval $(t-\theta, t)$. Since θ is the investment delay, all orders placed during the said interval are still unfilled, whereas all orders placed previously are already filled. Therefore we can write

$$W(t) = \int_{t-\theta}^{t} I(\tau)\, d\tau. \tag{3}$$

All the other economic relations between the investment indicators above are described in the publication [2].

Let us list of the most important equations related to the Kalecki's business model here.

- Delivery of finished capital goods *L* is dependent on the investment orders *I.* The progress of investment is the process realized in certain time interval and so we can write the equation

  $$L(t) = I(t-\theta)\,. \tag{4}$$

  This equation says that delivery of finished capital goods in time *t* is dependent on the investment order which is given in time $t-\theta$.
- Production of capital goods *A* is dependent on the investment order *I*. Since each order requires a period of time θ to be filled, and assuming that the construction of the order capital goods proceeds at an even pace (that is, $1/\theta$ of each order is executed per unit of time), it follows that the production of capital goods is equal to $(1/\theta)W$, therefore

  $$A = \frac{W}{\theta}\,. \tag{5}$$

  From (3) and (5) we have

  $$A(t) = \frac{1}{\theta}\int_{t-\theta}^{t} I(\tau)\, d\tau. \tag{6}$$

- If we call *K* the capital stock, its first derivative with respect to time is its net increment. This net increment of capital stock depends on the delivery of finished goods *L*, so that

  $$K'(t) = L(t) - U, \tag{7}$$

  where *U* indicates physical depreciation. Kalecki assumes that *U* is a constant.
- Investment orders *I* depend on the capital stock *K* and the production of capital goods *A*, therefore

  $$I = m(C_1 + A) - nK, \tag{8}$$

  where C_1, m, n are positive constants. *I* is increasing function with respect to the production of capital goods *A* because if the number of the investment orders *I* is increasing, the production of capital goods *A* is increasing too. On the other hand *I* is decreasing function with respect to the capital stock *K*.

Differentiating (6) and (8) we have

$$A'(t) = \frac{I(t) - I(t-\theta)}{\theta}, \tag{9}$$

$$I'(t) = mA'(t) - nK'(t). \tag{10}$$

From (4) and (7) we obtain

$$K'(t) = I(t-\theta) - U. \tag{11}$$

Substituting in (10) from (9) and from (11) we have

$$I'(t) = \frac{m}{\theta}\left[I(t) - I(t-\theta)\right] - n\left[I(t-\theta) - U\right]. \tag{12}$$

For simplicity we denote the difference $I(t)-U$ by $J(t)$ which represents net investment. Taking account of the fact that U is a constant, so that $J'(t)=I'(t)$, we have the equation

$$J'(t)-\frac{m}{\theta}J(t)+\left(\frac{m}{\theta}+n\right)J(t-\theta)=0. \tag{13}$$

The equation above is a delay differential equation of the form (2).

3. Existence of Positive Solutions

In the article [3] we study the existence of positive solutions of delay differential equation of the form (2). The main result is in the following theorem.

Theorem 1. *Suppose that there exist functions $k_1,k_2 \in C([t_0,\infty),(0,\infty))$ such that*

$$k_1(t)\le k_2(t), \quad t\ge t_0,$$

$$p(t)+k_1(t)q(t)\ge 0, \quad t\ge t_0,$$

$$\ln k_1(t)\le \int_{t-\theta}^{t}[p(s)+k_1(s)q(s)]ds \quad \textit{and} \quad \int_{t-\theta}^{t}[p(s)+k_2(s)q(s)]ds\le \ln k_2(t),\ t\ge t_0.$$

Then (2) has a solution which is bounded by positive functions.

With respect to (2) throughout we assume that $q\in C([t_0,\infty),[0,\infty))$, $q(t)\neq 0$, $p\in C([t_0,\infty),R)$, $\theta>0$.

From the proof of Theorem 1 [3] it follows that (2) has a positive solution which is bounded by functions

$$u(t)=\exp\left(-\int_{T}^{t}[p(s)+k_2(s)q(s)]ds\right), \quad v(t)=\exp\left(-\int_{T}^{t}[p(s)+k_1(s)q(s)]ds\right)$$

where $t\ge T\ge t_0+\theta$. Applying Theorem 1 to (13) we obtain:

Theorem 2. *Suppose that there exist functions $k_1,k_2 \in C([t_0,\infty),(0,\infty))$ such that*

$$k_1(t)\le k_2(t), \quad t\ge t_0, \tag{14}$$

$$-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k_1(t)\ge 0, \quad t\ge t_0, \tag{15}$$

$$\ln k_1(t)\le \int_{t-\theta}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k_1(s)\right]ds, \ \int_{t-\theta}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k_2(s)\right]ds\le \ln k_2(t),\ t\ge t_0. \tag{16}$$

Then (13) has a solution which is bounded by positive functions

$$u(t)=\exp\left(-\int_{T}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k_2(s)\right]ds\right), \quad v(t)=\exp\left(-\int_{T}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k_1(s)\right]ds\right)$$

where $t\ge T\ge t_0+\theta$.

For example if we take $k_1=1$, $k_2=2$, then conditions (14) and (15) are obviously satisfied and for conditions (16) we have

$$\ln 1=0\le \int_{t-\theta}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)\right]ds=n\theta, \ \int_{t-\theta}^{t}\left[-\frac{m}{\theta}+2\left(\frac{m}{\theta}+n\right)\right]ds=m+2n\theta\le \ln 2. \tag{17}$$

If we put m=n=0.25, θ=0.5, then conditions (17) are also satisfied and (13) has a solution which is bounded by the functions

$$u(t)=\exp\left(-\int_T^t\left(\frac{m}{\theta}+2n\right)ds\right)=e^{T-t},\ v(t)=\exp\left(-\int_T^t n\,ds\right)=e^{0.25(T-t)},\ t\ge T\ge t_0+\theta.$$

By [3] the next corollary is valid for Kalecki's business model.

Corollary 1. *Suppose that k>0 and*

$$-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k\ge 0,\quad t\ge t_0,$$

$$\int_{t-\theta}^{t}\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k\right]ds=\ln k,\quad t\ge T. \tag{18}$$

Then (13) has a solution

$$J(t)=\exp\left(-\int_T^t\left[-\frac{m}{\theta}+\left(\frac{m}{\theta}+n\right)k(s)\right]ds\right),\quad t\ge T.$$

From (18) we obtain

$$(m+\theta n)k=m+\ln k. \tag{19}$$

For $\theta=0.5$, $m=n=0.3$, (19) has two roots $k_1=1.37607$, $k_2=3.35961$ and *for* $t\ge T\ge t_0+\theta$ we have two net investment function

$$J_1(t)=\exp\left(-\int_T^t 0.63846\,ds\right)=e^{0.63846(T-t)},\ J_2(t)=\exp\left(-\int_T^t 2.42365\,ds\right)=e^{2.42365(T-t)}.$$

4. Conclusion

In the last several years the delay differential equations have been proposed as models for a variety of economic processes. Equation (13) represents Kalecki's business model and its solution *J(t)* is the net investment. The fundamental idea is, whether such equations can support the existence of positive solutions. Because by such solutions we can effectively observe the development of investment processes.

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Boundary-value Problem in an Economic Application

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Abstract. In the paper the theory of linear boundary-value problems is briefly summarized. As an application of this theory the market model is considered. This model is solved as a boundary-value problem and possible solutions are discussed.

Keywords: Boundary-value problem, criterion of solvability, quasi-solution, market model.

1. Introduction

Differential equations have become a vital part of mathematical modeling. Their aim is to mathematically describe the change, dynamics of the system.

Often the mathematical description of a system demands that we solve a non-homogenous linear differential system subject to boundary conditions – that is, conditions specified on the unknown function, or on one of its derivatives, or even on a combination of the unknown function and one of its derivatives, at two (or more) different points. Such systems are called boundary-value problems.

Wide range of boundary-value problems are used to model situations in physics and technology but nowadays they are finding applications in other fields of study as it will be shown in this article.

2. Boundary-value Problem

Let $A(t)$ is (n x n) matrix function, $f(t)$ is n-column vector function and elements of $A(t)$ and $f(t)$ are continuous on the interval $[a,b]$. Further, we will assume that $lz(\cdot)$ is linear bounded vector functional of the form

$$lz(\cdot) = \left(l_1 z(\cdot), l_2 z(\cdot), \ldots, l_m z(\cdot)\right)^T, \tag{1}$$

where

$$l_1 z(\cdot), l_2 z(\cdot), \ldots, l_m z(\cdot) : C[a,b] \to R \tag{2}$$

are linear bounded functionals.

Linear boundary-value problem can be defined as a system of ordinary linear differential equations

$$\frac{dz(t)}{dt} = A(t)z(t) + f(t) \tag{3}$$

which solution, that exists on the set of continuously differentiable functions on the interval $[a,b]$, satisfies boundary condition of the form

$$lz(\cdot) = \omega, \quad \omega \in R^m. \tag{4}$$

When solving boundary-value problem the existence of the solution is not guaranteed. For solution of (3), (4) to exist the following condition has to be satisfied

$$P_{Q^*}\{\omega - lK[f(s)](\cdot)\} = 0. \tag{5}$$

This equation is called the criterion of solvability of problem (3), (4). Here P_{Q^*} denotes orthoprojector of matrix Q^*, transposed matrix to the matrix Q. Matrix Q is defined as follows

$$Q = l\Phi(\cdot), \tag{6}$$

where $\Phi(t)$ is fundamental matrix of the linear part of system (3). Further, $K[f(s)](t)$ is called Green operator and is defined by the formula

$$K[f(s)](t) = \Phi(t)\int_a^t \Phi^{-1}(s) f(s) ds. \tag{7}$$

When (5) is satisfied, the solution of (3), (4) is in the form

$$z(t, c_r) = \Phi P_{Qr} c_r + \Phi Q^+ \{\omega - lK[f(s)](\cdot)\} + K[f(s)](t),\ c_r \in R^r. \tag{8}$$

Symbol P_{Qr} denotes (n x r) matrix consisting of r linearly independent columns of the orthoprojector P_Q. Symbol Q^+ denotes pseudoinverse matrix to the matrix Q.

On the other hand, when criterion of solvability is not satisfied, such problem is called ill-posed boundary-value problem. In this case, solution of (3) that satisfies (4) does not exist. But we could find the so-called quasi-solution. This solution represents only approximation of the solution of (3), (4). It is also of the form (8) and it could be shown that quasi-solution in this form minimizes (in the least-square sense) the difference

$$lz(\cdot) - \omega. \tag{9}$$

More on this subject could be found in [1], [2].

3. Dynamical Market Model

Now we will apply the before mentioned theory while solving the dynamical market model with one type of product.

Behaviour of the consumer is represented by the demand function

$$x = D(p), \tag{10}$$

where x denotes the amount of products and p denotes the unit price. Function (10) represents the amount of products that consumers are willing to buy for given price. It is obvious that with increasing price of the product the interest and therefore the bought amount of the product is decreasing.

Behaviour of the producer could be described by the supply function

$$x = S(p). \tag{11}$$

It represents the relationship between the price of the product and the amount of the products that the producer is willing to produce. The more money the producer earns for his product, the more he is willing to produce.

Demand and supply influence each other in the market. The situation when they are in balance is called the economic equilibrium. It is described by an equilibrium price p_E and an equilibrium quantity x_E. As the situation in the market is changing continuously the equilibrium is only temporary.

The development of price p during the time interval is described by the differential equation

$$\frac{dp(t)}{dt} = k[D(p) - S(p)], \tag{12}$$

where k denotes the sensitivity of price change in the market, $k > 0$. Equation (12) is called a dynamical market model for one type of product.

Now we will assume that both demand and supply are linear functions of the form

$$D(p): x = a - bp,$$
$$S(p): x = -c + dp, \tag{13}$$

respectively. Coefficients *a*, *b*, *c*, *d* are positive real constants. The equilibrium in the market is given by equality

$$D(p) = S(p) \tag{14}$$

and therefore from (13) and (14) we obtain the equilibrium price of the form

$$p_E = \frac{a+c}{b+d}. \tag{15}$$

The dynamical market model (12) will have the form

$$\frac{dp}{dt} = k[a + c - p(b+d)]. \tag{16}$$

It is obviously a non-homogenous linear differential equation. Its general solution is of the form

$$p(t) = C.e^{-k(b+d)t} + \frac{a+c}{b+d}, \quad C \in R. \tag{17}$$

Together with (16) we will consider the boundary conditions

$$lp(\cdot) = \begin{pmatrix} p(0) \\ p(1) \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}. \tag{18}$$

Fundamental matrix Φ of (16) is

$$\Phi(t) = e^{-k(b+d)t} \tag{19}$$

and if we denote $k(b+d)=q$, the fundamental matrix is of the form

$$\Phi(t) = e^{-qt}. \tag{20}$$

Matrix *Q* is

$$Q = l\Phi(\cdot) = \begin{pmatrix} 1 \\ e^{-q} \end{pmatrix} \tag{21}$$

and pseudoinverse matrix Q^+ is of the form

$$Q^+ = \frac{e^q}{e^{2q}+1}\begin{pmatrix} e^q & 1 \end{pmatrix}. \tag{22}$$

Orthoprojectors are

$$P_Q = E_1 - Q^+Q = 0,$$
$$P_{Q^*} = E_2 - QQ^+ = \frac{1}{e^{2q}+1}\begin{pmatrix} 1 & -e^q \\ -e^q & e^{2q} \end{pmatrix}. \tag{23}$$

Before finding the solution of (16), (18) we have to verify the criterion of solvability (5). First we express Green operator given by (7)

$$K[f(s)](t) = \left(1 - e^{-qt}\right)\frac{a+c}{b+d}. \tag{24}$$

Consequently, the solvability criterion formulated for dynamical market model (16), (18) is

$$P_{Q^*}\{\omega - lK[f(s)]\}(\cdot) = \frac{1}{e^{2q}+1}\begin{pmatrix} \alpha - \beta e^q + p_E(e^q - 1) \\ -\alpha e^q + \beta e^{2q} - p_E e^q (e^q - 1) \end{pmatrix}. \tag{25}$$

Here p_E denotes the equilibrium price given by (15).

If

$$\alpha = \beta e^{q} + p_E (e^{q} - 1) \tag{26}$$

then the solvability criterion is satisfied and there exists the solution of boundary-value problem (16), (18) in the form

$$p(t) = p_E - \left\{ p_E - \frac{e^{q}}{e^{2q} + 1} \left[\alpha e^{q} + \beta - p_E \left(1 - e^{-q} \right) \right] \right\} e^{-qt}. \tag{27}$$

On the other hand, if

$$\alpha \neq \beta e^{q} + p_E (e^{q} - 1) \tag{28}$$

then the boundary-value problem is ill-posed and the solution satisfying (18) does not exist. The quasi-solution of (16), (18) is in the form (27) and minimizes the difference (9).

4. Conclusion

Model discussed in this paper is an example of application of linear boundary-value problems. Linear boundary-value problems represent only the simplest type of boundary-value problems. Many fields of physics and technology are working with much more sophisticated types of boundary-value problems. Therefore methods for analyzing such problems are developing.

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On a Generalization of Hermite Polynomials

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Abstract. In this paper we study the system of generalized Hermite polynomials that could be derived from classical Hermite polynomials by a certain generalization of their weight function.

Keywords: generalized Hermite polynomial, reproducing kernel

1. Introduction

Orthogonal polynomial systems are very useful in many problems in the approximation theory, mathematical and numerical analysis, and their applications (Gaussian quadrature processes, least square approximation, differential and difference equations, Fourier series, etc.). Basic properties of orthogonal polynomials are described in [10, 11]. This paper was inspired by the famous works of J. Korous [3, 4]. Some other results about generalized orthogonal polynomials may be found in [5, 6, 9].

Let I be a finite or infinite interval and let the function $w: I \to \langle 0, \infty)$ be Lebesgue measurable with all power moments

$$\int_I x^n w(x)dx,\ n = 0,1,2,...$$

finite. Then $w(x)$ is called a weight function and can define orthonormal polynomials

$$p_n(x) = a_n^{(n)}x^n + a_{n-1}^{(n)}x^{n-1} + ... + a_0^{(n)},\ a_n^{(n)} > 0,$$

which satisfy

$$\int_I p_n(x)\,p_m(x)w(x)dx = \delta_{n,m},\ n,m = 0,1,2,...,$$

where $\delta_{n,m}$ is the Kronecker delta

$$\delta_{n,m} = \begin{cases} 1,\ n = m, \\ 0,\ n \neq m. \end{cases}$$

Let us define the reproducing kernel function by

$$K_n(x,t) = \sum_{i=0}^{n} p_i(x)\,p_i(t). \tag{1}$$

Using Christoffel-Darboux formula, this kernel can be written as

$$K_n(x,t) = \frac{a_n^{(n)}}{a_{n+1}^{(n+1)}} \cdot \frac{p_{n+1}(x)\,p_n(t) - p_n(x)\,p_{n+1}(t)}{x-t}.$$

Letting $t \to x$ we find

$$K_n(x,x) = \frac{a_n^{(n)}}{a_{n+1}^{(n+1)}} \cdot \left(p'_{n+1}(x)\,p_n(x) - p'_n(x)\,p_{n+1}(x)\right). \tag{2}$$

The function $K_n(x,t)$ plays a fundamental role in the integral representation of partial sums of the expansion of a function into the series of orthonormal polynomials. Now we need some properties

of the system of generalized Hermite polynomials $\{H_n(x)\}_{n=1}^{\infty}$, orthogonal in $I=(-\infty,\infty)$ with respect to the weight function

$$w(x)=|x|^{2\alpha}\,\mathrm{e}^{-x^2},\ \alpha>-\frac{1}{2}.$$

Because the interval I is symmetric and the weight function is even, there holds (cf. [10])

$$H_n(-x)=(-1)^n H_n(x).$$

It is obvious that

$$H_{2k+1}(0)=0, \tag{3}$$

$$\frac{d}{dx}H_{2k}(0)=0. \tag{4}$$

These polynomials have the form $H_n(x)=x^n+a_{n-1}^{(n)}x^{n-1}+...+a_0^{(n)}$, $a_n^{(n)}=1$. The system $\{\tilde{H}_n(x)\}_{n=0}^{\infty}$, where (cf. [1])

$$\tilde{H}_n(x)=\frac{1}{\gamma_n}H_n(x),$$

$$\gamma_n=\|H_n\|=\sqrt{(H_n,H_n)}=\sqrt{\int_I H_n^2(x)w(x)dx},$$

$$\gamma_{2k}^2=\frac{(2k)!!}{2^{2k}}(2k-1+2\alpha)(2k-3+2\alpha)...(1+2\alpha)\Gamma\left(\alpha+\frac{1}{2}\right),$$

$$\gamma_{2k+1}^2=\frac{(2k)!!}{2^{2k+1}}(2k+1+2\alpha)(2k-1+2\alpha)...(1+2\alpha)\Gamma\left(\alpha+\frac{1}{2}\right),\ k=0,1,2,...,$$

is orthonormal ($\Gamma(x)$ is a well-known gamma function, $(2k)!!=2k\cdot(2k-2)\cdot...\cdot4\cdot2$). The classical Hermite polynomials are the special case of the polynomials $\tilde{H}_n(x)$ for $\alpha=0$. Based on the above considerations and (2), we state

$$K_{2k}(x,x)=\sqrt{\frac{2k+1+2\alpha}{2}}\left(\tilde{H}'_{2k+1}(x)\tilde{H}_{2k}(x)-\tilde{H}'_{2k}(x)\tilde{H}_{2k+1}(x)\right), \tag{5}$$

$$K_{2k+1}(x,x)=\sqrt{k+1}\left(\tilde{H}'_{2k+2}(x)\tilde{H}_{2k+1}(x)-\tilde{H}'_{2k+1}(x)\tilde{H}_{2k+2}(x)\right). \tag{6}$$

Now we use the forms of the shift operators (cf. [1])

$$\frac{d}{dx}\tilde{H}_{2k}(x)=2\sqrt{k}\tilde{H}_{2k-1}(x),$$

$$\frac{d}{dx}\tilde{H}_{2k+1}(x)=-\frac{2\alpha}{x}\tilde{H}_{2k+1}(x)+2\sqrt{\frac{2k+1+2\alpha}{2}}\tilde{H}_{2k}(x),$$

$$\frac{d}{dx}\tilde{H}_{2k+1}(0)=\frac{2}{1+2\alpha}\sqrt{\frac{2k+1+2\alpha}{2}}\tilde{H}_{2k}(0).$$

Using these operators and (5), (6) we have reproducing kernel functions

$$K_{2k}(x,x)=$$

$$=\sqrt{\frac{2k+1+2\alpha}{2}}\left[-\frac{2\alpha}{x}\tilde{H}_{2k+1}(x)\tilde{H}_{2k}(x)+2\sqrt{\frac{2k+1+2\alpha}{2}}\tilde{H}_{2k}^2(x)-2\sqrt{k}\tilde{H}_{2k-1}(x)\tilde{H}_{2k+1}(x)\right], \tag{7}$$

$$K_{2k+1}(x,x)=\sqrt{k+1}\left[2\sqrt{k+1}\tilde{H}_{2k+1}^2(x)+\frac{2\alpha}{x}\tilde{H}_{2k+1}(x)\tilde{H}_{2k+2}(x)-2\sqrt{\frac{2k+1+2\alpha}{2}}\tilde{H}_{2k}(x)\tilde{H}_{2k+2}(x)\right]. \tag{8}$$

From (1) and (3) we get a special case for $x=0$

$$K_{2k}(0,0)=\sum_{i=0}^{2k}\tilde{H}_i^2(0)=\sum_{i=0}^{2k+1}\tilde{H}_i^2(0)=K_{2k+1}(0,0)$$

and with regard to (4) we obtain

$$K_{2k}(0,0)=K_{2k+1}(0,0)=\frac{2k+1+2\alpha}{1+2\alpha}\tilde{H}_{2k}^2(0). \tag{9}$$

2. Main Results

Using the previous results, we can formulate the following theorem.

Theorem 1. Let $\left\{\tilde{H}_n(x)\right\}_{n=1}^{\infty}$ be the system of polynomials orthonormal in the interval $I=(-\infty,\infty)$ with respect to the weight function $w(x)=|x|^{2\alpha}\,\mathrm{e}^{-x^2}$, $\alpha>-\frac{1}{2}$. Let $\left\{h_n(x)\right\}_{n=1}^{\infty}$ be the system of orthonormal polynomials associated with the weight function $W(x)=h(x)\cdot|x|^{2\alpha}\,\mathrm{e}^{-x^2}$ in the same interval I, where the function $h(x)$ satisfies the condition

$$0<m\le h(x),\ x\in I.$$

Then we have the following estimations

$$|h_{2k}(x)|\le\sqrt{\frac{2k+1}{m}}\sqrt{K_{2k}(x,x)},$$

$$|h_{2k+1}(x)|\le\sqrt{\frac{2k+2}{m}}\sqrt{K_{2k+1}(x,x)},$$

$$|h_{2k}(0)|\le\sqrt{\frac{2k+1}{m}}\sqrt{\frac{2k+1+2\alpha}{1+2\alpha}}\left|\tilde{H}_{2k}(0)\right|,$$

$$|h_{2k+1}(0)|\le\sqrt{\frac{2k+2}{m}}\sqrt{\frac{2k+1+2\alpha}{1+2\alpha}}\left|\tilde{H}_{2k}(0)\right|,$$

where the kernels are given by (7) and (8).

Proof. Every $h_{2k}(x)$, $h_{2k+1}(x)$ can be uniquely represented as a linear combination of the system $\left\{\tilde{H}_n(x)\right\}_{n=0}^{\infty}$

$$h_{2k}(x)=\sum_{i=0}^{2k}c_i\tilde{H}_i(x), \tag{10}$$

$$h_{2k+1}(x)=\sum_{i=0}^{2k+1}c_i\tilde{H}_i(x), \tag{11}$$

where the Fourier coefficients are given by

$$c_i=\int_I h_r(t)\tilde{H}_i(t)w(t)dt$$

and $r=2k$ or $r=2k+1$.

Coefficients c_i may be written in the form

$$c_i = \int_I h_r(t)\tilde{H}_i(t)w(t)dt = \int_I h_r(t)\tilde{H}_i(t)w(t)\sqrt{\frac{h(x)}{h(x)}}dt,$$

and applying Schwarz inequality and the orthonormal properties of the polynomials $\tilde{H}_n(x)$, $h_n(x)$ we get

$$|c_i| \le \frac{1}{\sqrt{m}}\int_I |h_r(t)|\cdot|\tilde{H}_i(t)|\,w(t)\sqrt{h(x)}dt \le \frac{1}{\sqrt{m}}\sqrt{\int_I \tilde{H}_i^2(t)w(t)dt}\sqrt{\int_I h_r^2(t)h(t)w(t)dt} = \frac{1}{\sqrt{m}}.$$

Now we use (10), (11) and by Cauchy inequality we have

$$h_{2k}^2(x) = \left(\sum_{i=0}^{2k} c_i\tilde{H}_i(x)\right)^2 \le \sum_{i=0}^{2k} c_i^2 \sum_{i=0}^{2k}\tilde{H}_i^2(x) = (\sum_{i=0}^{2k} c_i^2)K_{2k}(x,x),$$

$$h_{2k+1}^2(x) = \left(\sum_{i=0}^{2k+1} c_i\tilde{H}_i(x)\right)^2 \le \sum_{i=0}^{2k+1} c_i^2 \sum_{i=0}^{2k+1}\tilde{H}_i^2(x) = (\sum_{i=0}^{2k+1} c_i^2)K_{2k+1}(x,x).$$

Finally, using the inequalities $|c_i| \le \frac{1}{\sqrt{m}}$ and (9), we get the statement of the theorem.

3. Conclusion

In this paper we studied generalized Hermite polynomials of a certain type. Our investigations are based on our previous results reached in [1]. Some other results on classical and generalized Hermite polynomials can be found in [2, 7, 8].

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Some Issues Pertaining to the Cultural and Natural World Heritage. Specifying its Scope, Purpose and Selected Contrast Aspects.

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Abstract. Most countries all over the globe possess cultural or natural places of unique and extraordinary values. These places are usually of great historical, scientific, aesthetic, ecological and environmental importance. The most valuable of them have already been listed to UNESCO World Heritage Site after recommendation of competent experts and international committees. Slovakia as an EU country also plays an important role in World Heritage conservancy programmes by means of appending its own unique and outstanding cultural and natural sites to the World Heritage list and by promoting various conservation and scientific programmes within these fields. The aim of the paper is to make a short note on World Heritage Sites as a common property of the whole mankind. In the paper I deal with scope and purpose of World Heritage. Particular attention is also placed on different aspects within Heritage sites and characteristics of natural heritage sites.

Keywords: World heritage, conservancy, scope, threats, cultural, natural, sites.

1. Introduction

There is no doubt that World heritage sites are the most valuable places around the globe from the wide variety of spectrum. Because of their uniqueness, beauty and immeasurable values these places have become a legacy of mankind as a whole. Their benefits cannot be calculated, these places are sources of ancient testimony and knowledge, they fulfil our spirit and soul with science and art, within its extraordinary beauty we find inner peace and rest, they make us better man. Many of these places have also become great attractions and play a significant role in tourist industry.

Unfortunately, the World heritage sites are constantly being in danger because of the many "natural" and "cultural" threats undermining their status and values. Therefore, it is an imperative for all of us to do the utmost to preserve these sites for present and future generations. Sometimes it is a "fight" against "ravages of time" and sometimes again "society development" and sometimes just against our own idleness and selfishness.

In general, within the World Heritage property we distinguish between cultural and natural elements. Each category is specific and contributes evenly to the world heritage its own way. First of all, it is important to realize their outstanding values, their purpose to local, national and international community, the need to preserve these irreplaceable sites and the availability of conservancy methods. Consequently it is important to realize our own involvement and attitude to World heritage conservancy on local or regional scale e.g. by promoting presentation, research or education programmes.

2. Semantics and Origins of „heritage"

According to online etymology dictionary a term „heritage" is derived from Old French „héritage" and Latin „heredis". [1] Both these languages have played an important role in English language development processes. Reviewing modern English dictionaries e.g. Oxford , Cobuild, Collins and Macmillan we can find several synonyms of "Heritage". The closest of them are e.g

„inheritance“ and „legacy“ which also come from Old French and Medieval Latin. [1] The linking idea among all these synonyms mean that there exit something valuable and precious that is worthy keeping also for next generations. However, from a more detailed semantic point of view words like „Heritage“ and its closest synonyms such as „inheritance“ and „legacy“ have two distinct denotations.

The first meaning has a narrower impact and deals with something that comes or belongs to one by reason of birth, something that has been or may be inherited by legal descant or succession. It may be any property, especially land that devolves by right of inheritance. [2]

The second meaning has much wider impact and refers to the all qualities, cultural traditions or features of society‘ life that have been continued over many years and passed on from one generation to another, especially ones that are of historical importance, value or that have had a strong influence on society. [3]

3. International Organisations Performing World Heritage Conservation

For the sake of “World Heritage” conservancy many national and international principles, recommendations and charters have been introduced mainly due to the incomplete protection of world heritage at the national or regional levels. In general, their prime role is to register, preserve and present cultural and natural property of exceptional importance across the globe. Many of these principles and declarations have been introduced and adopted by international organisations such as UNESCO, ICOMOS and IUCN. All these organisations are professional associations that also focus on the conservation and protection of cultural and natural heritage places around the world.

UNESCO (The United Nations Educational, Scientific and Cultural Organization) is a specialized organisation of the United Nations (UN). Its prime purpose is to promote education, science and culture and thus secure peace in many countries of the world. It pursues its objectives through five major programs: education, natural science, social and human science, culture, and communication and information [4]. There are many projects sponsored by UNESCO including global literacy support , various teacher-training programmes, international science programmes, regional and cultural history projects, promotion of cultural diversity and agreements to secure the world cultural and natural Heritage (World Heritage Sites).

ICOMOS (International Council on Monuments and Sites) was founded in 1965 in Warsaw as a result of the Venice Charter of 1964 and offers advice to UNESCO on World Heritage Sites [5].

IUCN (International Union for Conservation of Nature and Natural Resources) is one of the most important global nature conservation Instruments, besides other nature conservancy activities it coordinates The World Heritage Programme with focus on natural heritage conservancy. This organisation was founded in 1948 as the world’s first global environmental organization [6]. Currently, it is a leading authority on the environment and sustainable development issues.

The most significant achievements of heritage conservation activities were taken in the second half of the 20th century when crucial conservation principles were developed and ratified. These main conservation events were Venice Charter in 1964 and World Heritage Convention in Paris in 1972 [7].

4. Purpose of World Heritage

Throughout cultural (society) or natural (habitat) development processes, which usually take many centuries or even millenniums, many extraordinary and outstanding values might be created or invented by man crafts and skills or nature laws itself. On the other hand, sometimes a lot of efforts have to be taken not to destroy something that have already been created by ancestors or have developed on its own on the grounds of physical and natural laws.

It does not really matter whether these values have been created by man or have originated by its own forces , they together represent a irreplaceable property of mankind – „World heritage“ sites and places where man's life can be filled with knowledge, inspiration, enrichment, joy and health. Whether, it is an architecture construction, a piece of art, a ruined medieval castle, primeval forest or a nature reserve they are all a common mankind property.

We are all very well aware that the cultural heritage and the natural heritage are increasingly threatened with destruction not only by the traditional causes of dilapidation and decay, but also by life style alternations and many developing activities aggravated by modern phenomena, or by transformation of economic and social conditions and finally by decline or extinction of traditional handcrafts and techniques. Therefore, it is an imperative for all of us to guarantee the Heritage conservation on both international and national legislative levels. Only this way we can secure that all these values of the present and the past will be also shared with next generations. Destruction and devastation of these values and property will undoubtedly mean irreparable loss for the mankind as a whole.

5. Tangible and Intangible Scope of World Heritage

At present time, the scope of Heritage include „tangible“ and „intangible“ values or property forming both the cultural and natural part of World Heritage. In more detailed statements Cultural property includes the physical, or "tangible" cultural heritage, such as buildings and historic places, monuments , books, documents, works of art, machines, clothing, and other artefacts, that are considered worthy of preservation for the future. „Intangible“ cultural heritage includes such as folklore, traditions, language, and knowledge.

Besides Cultural Heritage that is more associated with human creativity, craft and imagination, there is also other part of World Heritage, a part that is more associated with wildlife and biodiversity – Natural Heritage. This part is also an important part of a society's life, encompassing the countryside and natural environment, including flora and fauna, scientifically known as biodiversity, as well as geological elements (including mineralogical, geomorphologic, paleontological, etc.), scientifically known as geodiversity. [8] Natural heritage sites, natural reserves or in general any natural sites of outstanding values often serve as an important component in a society's life e.g. scientific research, resource of inspiration, relax, physical and psychical health, sport activities and tourist industry. These sites usually attract many researchers and visitors either from abroad or locally. Natural Heritage can also include cultural landscapes (natural features that may have cultural attributes).

6. Some Contrast Aspects within Cultural and Natural Heritage

World heritage encompasses both cultural and natural sites. Their common denominator is the outstanding or extraordinary feature. But there is also a contrast, something that makes a difference within this outstanding property of mankind. The contrast can be easily noticed from the very substance of the terms „Cultural“ and „Natural“. A term „Cultural“ pertain to „culture“ or „cultivation“ and thus is deeply intertwined with man's life and his corresponding activities. A term „Natural“ pertain to „nature“ itself, meaning that something has been formed by nature as opposite to „artificial“ activities. Natural things exist or occur in nature and are not made or caused by people. This is also a fundamental factor of „driving forces“ which is responsible for fashion of creation and we can spot herein the very difference between these two parts of World heritage.

However, there is also other significant contrast between terms „cultural“ and „natural“ which might draw our interest. We are all aware that the cultural heritage and the natural heritage are constantly threatened with destruction. From these reasons, most of World Heritage programmes are focused on conservation efforts, protecting this valuable property against various kinds of

threats. There are many factors threatening both cultural and natural elements of World Heritage equally e.g. social and economic development or abandonment of traditional values. But there is one threat that makes a difference. This threat is called „the ravages of time“ and its factor plays an important role in distinguishing between „cultural“ and „natural“ property.

In cultural heritage, where the driving forces have always been associated with human activities with all his creative imagination, „the ravages of time“ are one of the most acute factor threatening this cultural property and thus are an enemy No. 1.

In conservation of natural heritage the situation is diametrically opposite. Driving forces are not of human origins but it is nature itself with its all physical and natural laws. The ravages of time are crucial and inevitable factor in creation of intact valuable habitats. The ravages of time are not an enemy but a crucial biodiversity factor that is inevitable for quality habitat development. Not letting “the ravages of time” do its job properly in natural habitats leads from ecological and biological point of view to grief situation in ecosystem formations. Other Natural Heritage “enemies” are mostly associated with human activities e.g. logging, hunting, and even unsustainable tourist industry or sport centre developments.

7. Slovakia Involvement in UNESCO, ICOMOS an IUCN Activities

Slovakia is a geographically small country, but very rich in both cultural and natural heritage. From these reasons Slovakia is involved in many activities of UNESCO, ICOMOS and IUCN to secure more effective conservancy systems. Some of the most important activities of the recent times are: the Memory of the World Programme and UNESCO Programe Man and the Biosphere.

The Memory of the World Programme was established by UNESCO in 1992 to facilitate preservation and access to documentary heritage in various parts of the world. The objectives of the Memory of the World are that the world's documentary heritage belongs to all, should be fully preserved and protected for all and, with due recognition of cultural mores and practicalities, should be permanently accessible to all without hindrance. This can be achieved by three steps: facilitating preservation of the world's documentary heritage, providing universal access to documentary heritage and increase of awareness worldwide of the existence and significance of documentary heritage. [9]

The most significant documentary heritage submitted by Slovakia and recommended for inclusion in the Memory of the World Register are **Illuminated Codices from the Library of the Bratislava Chapter House and Mining maps and plans of the Main Chamber**. The first phase of the Illuminated Codices project included the compilation of a CD-ROM containing the rare Antiphonary of Bratislava II, thus creating the basis for the digital edition of a collection of codices of Slovak provenance. Making the digitized documents accessible on CD-ROM enabled the dissemination of information on, and study of, the Antiphonary in great detail while preserving the original. [10]

The documentary heritage Mining maps and plans of the Main Chamber in Banská Štiavnica represents a collection of almost 20 thousand of items from the 17th century up to the early years of the 20th century, which are an essential part of the archival fonds of this the most significant authority for mining, metallurgy and minting in the Hungarian part of the Austro-Hungarian Empire. The maps and plans display underground mining works as well as above-ground objects in all the mining regions of Slovakia. [11]

Slovakia is also very rich in biodiversity. Mountain areas in Slovakia comprise about three fifths of the territory. In Slovakia there are nine National Parks and 14 Landscape Protected Areas. Three National Parks and one Landscape Protected Area have already been included in the **UNESCO Programme Man and the Bioshpere** (MAB). The Man and the Biosphere Programme is an Intergovernmental Scientific Programme aiming to set a scientific basis for the improvement of the relationships between people and their environment. Biosphere reserves are sites established by countries and recognized under UNESCO's Man and the Bioshpere Programme to promote

sustainable development based on local community efforts and sound science. After their designation, biosphere reserves remain under national sovereign jurisdiction, yet they share their experience and ideas nationally, regionally and internationally within the World Network of Biosphere Reserves. Biosphere reserves are thus globally considered as sites of excellence where new and optimal practices to manage nature and human activities are tested and demonstrated.

IUCN Members in Slovakia are Slovak ecological Society, State Nature Conservancy of the Slovak Republic, Association of National Parks and Protected Areas of Slovakia. IUCN Slovakia is a non-governmental organization, the aim of which is to share the promotion of IUCN mission, which is to influence, encourage and assist societies throughout the world to conserve the integrity and diversity of nature and to ensure that any use of natural resources is equitable and ecologically sustainable. [12]

The mission, objectives and activities of IUCN Slovakia are demonstrated by several successful projects, nature conservation activities in several landscape protected areas and a proposal of the national ecological network (NECONET) of Slovakia. They serve as reserves of richness of biodiversity and function as eco-stabilising factors of the landscape, mutually interconnected by ecological corridors. [13]

8. World Heritage UNESCO in Slovakia

In Slovakia there are many cultural and natural properties of outstanding value. Like others, they are irreplaceable resources of testimony, knowledge, science and emotions in development of society in this country. The most outstanding of them, have already been ascribed with the attribute of World Heritage Sites. In fact, five cultural and two natural sites have been listed in Slovakia so far.

The aim of the UNESCO programme is to register, name, conserve and present sites of the most outstanding cultural or natural importance. As of March 2012, there were 936 World Heritage Sites: 725 cultural, 183 natural, and 28 mixed properties, in 153 countries. [14] Any important and outstanding site of natural heritage or cultural heritage can be added to this World heritage Site by the World heritage Committee of UNESCO after complying with UNESCO World Heritage policy.

Some of the cultural ones are Historic town of Banska Stiavnica with the technical monuments (1993), a reserve of intact rural settlement in Vlkolinec with its architectural (1993) and Wooden churches of the Slovak part of the Carpathian mountain area (2008). The natural ones are Caves of Slovak Karst and Aggteleg Karst (1995) and Beech primeval forest of the Carpathians in NP Poloniny and PLA Vihorlat (2007). [15]

Besides these seven top sites there are other cultural and natural sites of outstanding value waiting for experts or committees to discover them and to recognise their values for national or international community and finally by doing this to secure their stability. Some of these sites have already been submitted to the Tentative List, what is a preceding process in being eventually enlisted to the World heritage Sites. There are 14 properties on the tentative list in Slovakia. Some of the cultural ones are: Historical town core of Košice City, Sites of Great Moravia at Mikulčice and Gemer and Abov churches. Some of the natural ones are: Natural reserves of Tatras Mountain, Fungal Flora of Bukovská Hills and Karst Valleys of Slovakia. [16].

9. Conclusion

Slovakia is one of the countries that have already been listed to UNESCO world Heritage sites because of its outstanding and exceptional cultural and natural values. So far, there have been seven sites listed to the UNESCO World Heritage Site. These places have been chosen because of its cultural or natural uniqueness. However, there are other exceptional places in Slovakia with historical, aesthetic, scientific or environmental importance to local, national or international

community. Some of them have already been added to the tentative list and others are just waiting to be disclosed and recognised and thus bestowed with the proper conservancy they deserve. It is not an easy task nowadays because the rules are made by profit makers. So to conserve e.g. a nature reserve or a national park that will result in better environmental, scientific or aesthetic values to all of us is usually a battle with developing investors that already have other plans that bring sole financial benefit to small chosen groups. But I hope that over a time majority of humankind will realize the importance of these sites and contrive such social pressure that these sites will be eventually granted by efficient conservancy programmes and policy. Today, we can only hope that "tomorrow" will not be late. Therefore it is an imperative to act now. The developers, new values and styles, the "ravages of time" do not wait but proceed to acquire their aim.

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Quality and Quantity Methods Used to Document Some Aspects of Species Conservation as a Tool of Natural Heritage Programmes.

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Abstract. Natural heritage is an important part of modern society. To secure efficient conservation of valuable natural areas and its inhabitants (diverse protected species) a lot of efforts have to be taken beforehand. The first step is to record the current status of selected natural habitats and important species. This paper summarizes accessible quality and quantity methods used for monitoring species and their key population data. Subsequently these data are used to implement efficient conservation systems in national or international heritage programmes, supporting conservation efforts in territory (habitat) or species protection.

Keywords: Natural heritage, nature conservancy, ecology, population, species, method.

1. Introduction

Both Cultural and Natural Heritage represent the most valuable monuments or sites over the globe from various objective reasons. Like Cultural Heritage, Natural Heritage is also an important part of a society's life because natural monuments or natural sites can often serve as an important component in a society's life e.g. science, research, resource of inspirations, relax, physical and psychical health, sport activities and tourist industry.

Natural heritage encompasses countryside and natural environment, including flora and fauna, scientifically known as biodiversity, as well as geological elements including mineralogical, geomorphologic and paleontological, scientifically known as geodiversity. Natural Heritage can also include cultural landscapes, or natural features that may have cultural attributes. [1]

Both Cultural and Natural heritage is increasingly threatened with various destruction forces forged by life style alternations aggravated by modern phenomena, or by transformation of economic and social conditions and finally by decline or extinction of traditional lifestyle, handcrafts and techniques. [2]

To save these values from further devastation a lot of human efforts must have been employed as well as various conservation systems have been introduced to declare and maintain the most valuable natural monuments and areas. In Slovakia, two natural sites have already been listed to the UNESCO World Heritage Site. [3] Other natural areas, phenomena and monuments, not less valuable, have become a national heritage. All such monuments or areas represent the most valuable Slovak property from environmental or ecological point of view and therefore under protection of Slovak legislative and international treaties like Bern Convention, CITES, IUCN and Ramsar Convention. Slovak natural Heritage property has its own system of recognition, categories and conservancy.

Thanks to The Carpathians, Slovakia is a country abundantly rich in natural monuments, extraordinary landscapes, beautiful sceneries and home for viable populations of endemic, rare and threatened species. Some of these species are very important in European continent and indeed deserve to become a national property and a part of natural heritage programmes. Although, probably there are nowadays no genuine natural forests and woodlands, yet there are some areas which throughout conservancy efforts have returned to their original function and status.

The conservancy system of natural heritage dates back to the late 19th century and has a long and distinguished history in Slovakia. The conservancy system has developed correspondingly with society development, urban expansion, growth of tourist industry, ascend of environmental needs, design of natural areas and increasing human pressure on protected areas.

Basically all conservancy systems all over the world consist of territory (landscape/habitat) conservancy legislative and species conservancy legislative. In Slovakia, the basic conservancy is secured by the Slovak Constitution which sets responsibility to all citizens to protect the environment and protect selected living entities. Important role is also played by national legislative issued by ministry of environment and several international conventions. Territory conservancy in Slovakia uses the system of five levels conservancy. In this system the most valuable areas are granted the highest level of protection that restricts any human interference within the area. Species conservancy uses the current status of the species within a region or a country as a key factor setting an appropriate level of protection. To set an appropriate level of protection many measures have to be taken into account beforehand. Thus every category (territory or species conservancy) has its own apt methods and techniques to measure the status. In category of territory or landscape conservancy we measure the intactness of the area, its biodiversity, compactness and so on. In category of species conservancy to acquire the status data we measure basic ecological species parameters e.g.: distribution, abundance, density, social structure, population trend, genetic efficiency, homerange, activity patterns and so on. To measure these various ecological or other environmental parameters we have to use an appropriate method or technique.

2. Theoretical Definitions and General Characteristics of Applied Techniques

Diverse methods have been introduced in Nature Conservation, ecology and Natural Heritage programmes to secure either the habitat or species conservancy programmes. Their development and innovations have been surged by increasing need for more efficient species management and better conservation efforts. However, not all methods can be efficiently applied in every ecosystem, environmental conditions and for all species. The use of particular method is therefore determined by several factors: research objectives, environmental conditions and field costs.

Three broad categories of monitoring techniques can be recognised, each with increasing levels of fieldwork required. The first category includes those techniques that do not require original fieldwork (questionnaires and reports). The second category involves fieldwork, but where individually recognisable individuals are not available (sign surveys and line transects). The third category includes methods where fieldwork has recognisable individuals available (camera traps and genetic methods). [4] From the invasive point of view these methods are divided into two broad categories. Invasive methods require certain degree of interference and disturbance of researched species, whereas non-invasive methods are more ”human” and do not require trapping or disturbing the animal.

The results of these methods provide researchers with data that are crucial for the proper conservation and management of species (population). From these data we might determine the appropriate level of protection or we can measure the success or failure of management strategies. Repeated estimates of population size are vital to determine if the population is decreasing, increasing or stable and thus tell us about population trend. Population data or population estimates are usually difficult to obtain, especially for rare, elusive and wide homerange species (e.g. Ursus arctos), therefore to obtain a certain level of accuracy a large amount of fieldwork and laboratory works are required.

3. Methods Used in Species Conservancy and Management of Species

3.1. Line Transects

A transect is a path along which one counts and records occurrences of the phenomena of study (e.g. plants, footprints crossing the trail, acoustic signs etc.). It requires an observer to move along a fixed path and to count occurrences along the path and, at the same time, obtain the distance of the object from the path. Very often this procedure is best performed soon after snowfall to record fresh tracks and help to eliminate the old tracks of the same individual. Tracking animals by following footprints in dust, mud, sand or snow, is probably the oldest known method of identifying mammal's presence in an area. Counts of dung, nests, trails, calls and direct observation along line transects are also widely used for richness and abundance estimates. [5]

Method of line transects is also used in other survey techniques e.g.: Whole Range Census or in Winter Route Census. The Winter Route Census (WRC) was suggested as a primary uniform technique for determining densities and population sizes of diverse species throughout Russia. [6]

Method of line transect is used to indicate some of the basic ecological data on monitored species. Using this method we can acquire data on distribution (presence – absence status), population size, population structure, densities, habitat suitability etc. The estimation of the abundance of populations (such as terrestrial mammal species) can be achieved using a number of different types of transect methods, such as strip transects, line transects, belt transects, curved line transects and triangle transects. Subsequently, density of each species can be calculated by number tracks per km of trail divided by the length of average day route of particular species

From the time perspective we can use this method all year round to obtain an annual data on species distribution, densities, population sizes and population structures. The method of line transects is also used in whole range census that usually takes two days to survey area for species presence. Method of line transects is efficient and usually involve low costs, but depend on suitable field conditions and trained personnel.

3.2. Camera Trapping Methods

Camera trapping refers to the use of remotely triggered cameras that automatically take images of whatever walks in front of them. Most camera models are triggered by a passive infrared sensor detecting a moving object warmer than the ambient temperature such as animals or people passing in front of them. Trail cameras usually consist of a small, camouflaged and waterproofed box with a lens. Works in a similar principle to a digital camera, but with added features like motion detection, night-vision, timed recording. Camera trapping is most often used to capture images of medium to large sized terrestrial mammals and birds, but has also been recently used for arboreal mammals. [7] Camera trapping methodology underwent significant advances and has been increasingly used in the last decade. [8]

The primary objective of camera trapping methodology is to survey populations and estimate several species and population data e.g.: distribution, population size, population structure, densities, activity patterns and other behaviour aspects. The method is very efficient for inventories, especially of cryptic animals, as well as for population studies of species for which individuals can be individually recognized by marks. [9]

Camera trapping is a non-invasive method that generally causes a minimum of disturbance to the target species. Camera traps can be left unattended in the field for several months, and thus are ideally suited for studying rare, elusive, and nocturnal (crepuscular) animals that avoid humans.

Camera trapping was invented in the late 1890s by George Shiras III, a Yale educated lawyer who perfected a way of photographing wildlife at night with a large-format camera and hand-operated flash. [10] From 1990s camera traps started being used in major research projects all over the world. Thanks to many advances, camera trapping has become a widely used tool in wildlife biology, opening the way to an impressive number of studies. The method of camera trapping is

more costly at the beginning but is not so dependent on the environment conditions, constant assistance or even experienced field staff.

3.3. Satellite Telemetry

In a broad sense telemetry is the highly automated communications process by which measurements are made and other data collected at remote or inaccessible points and transmitted to receiving equipment for monitoring. [11]

In Nature Conservation, ecology and Natural Heritage programmes Satellite telemetry is a researched method used to track important ecological information and behaviour of species. By using satellite tags, we can learn important information such as habitat use, activity pattern, behaviour and home range that can help in species and population conservation efforts. Lot of information in population biology is obtained from research facilities and captivity, but satellite telemetry is a tool to study species in the wild and thus can provide us with genuine information in real environment.

The disadvantage of this method is its invasive character and cost. Any researched animal has to be first anaesthetized, and then fitted with radio-collars before being re-released to the wild.

Once the animal is fitted with a radio-collar this method can be further characterized as a non-invasive method, meaning that data are collected for a long period without disturbing the individual animal. All collected data can be subsequently used in better conservancy efforts, more efficient management of species and improved understanding of studied individuals.

3.4. Method of Genetic Sampling

Genetic techniques have received much interest in the last few years. Because each individual is characterized by a unique multilocus genotype, DNA from remotely collected samples of hair or faeces provides a means of assessing attributes of populations of wild animals, including determination of sex, individual identity, genetic diversity, spatial distribution, patterns of habitat use, dispersal distances, population fragmentation and through the use of statistical models (capture-mark-recapture-methods), to estimate population size.

It is a noninvasive technique because it doesn’t require capture of studied animals and genetic sampling (i.e., hairs or faeces) are easily collected even without the need to see or disturb the animal. The genetic methods use Polymerase chain reaction (PCR) to amplify low quantities of DNA extracted from available samples (usually faeces and hair).

However, the amplification of DNA extracted from noninvasive samples is often problematic because the DNA is usually degraded and present in low quantity. This can often lead to scoring errors such as allelic dropout or false alleles and will often produce incorrect genotypes. [12] To avoid biased results improved genotyping methods (multiple tubes approach) have been proposed to limit genotyping errors and their impacts on the subsequent analysis. The disadvantages of these methods are the costs and that this method is suitable only for small population living in a closed environment.

3.5. Mark and Recapture Method

This method is commonly used in ecology to estimate population size and is most valuable when a researcher fails to detect all individuals present within a population of interest every time that researcher visits the study area. Researcher visits a study area and uses a technique (traps, camera traps or DNA traps) to capture a group of individuals alive. Each of these individuals is marked with a unique identifier and then is released unharmed back into the environment. Sufficient time is then allowed to pass for the marked individuals to redistribute themselves among the unmarked population. Next, the researcher returns and captures another sample of individuals. Some of the individuals in the second sample have been marked during initial visit and are now

know as recaptures. So population size can be estimated from as few as two visits to the study area. However, this method assumes that the study population is "closed".

This method is summed up in the Lincoln- Petersen formulae (also known as the Lincoln Index). [13]

$$N \frac{(M+1)(C+1)(M-R)(C-R)}{(R+1)(R+1)(R+2)}$$

Where
N = estimated of total population size
M = total number of animals captured and marked on the first visit
C = Total number of animals captured on the second visit
R = Number of animals captured on the first visit that were then recaptured on the second visit

4. Conclusion

According to the national legislative and several international conventions we are obliged to consider certain habitats (areas) and its selected inhabitants (species occupying the territory) to be very important from the ecological or environmental point of view. Due to their importance and status many of them have become a property of national or international heritage programmes. Nature itself is a living and evolving entity. To grand it efficient conservancy we must be aware of all its complexness, relations and consequences. Habitat quality reflects population (species) quality and vice versa. It is therefore crucial to treat them as a coherent unit and bestow them with efficient mutual protection. At the beginning is always important to know their real status and understand their needs. We can do that by using diverse noninvasive methods that are available at present time.

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The Effect of Printing and Fonts on the Result of Digitalization Process

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Abstract. The contribution deals research of the efect of printing and fonts on the result of digitalization process which is the topic os my dissertation paper and the assumption that language and type of fonts have key importance to the effectiveness and quality of digital content. It analyzes the process of research to confirm the hypothesis and the steps that lead to the creation of a integrated sampler of fonts as the main output of the work.

Keywords: Digitalization, digital content, language, font, typology of font, printer and printers, Slovakia-related documents.

1. Introduction

Nowadays when the digitalization of documents is very current topic. The digitalization of our cultural and written heritage is active. Area of digitalization are cultural objects which justify the development, current status and functioning of the Slovak language, libraries' collections (newspapers, magazines, books, music, cartographic materials, etc..), Historical library documents and funds, academic, final, qualifying work (ETD) , collections of museums and galleries, documents and creations of science, technology, movable and immovable monuments, archaeological monuments, archival documents, monument fund, traditional folk culture, art of music, literature, drama, dance and stage design, art film, photography, audiovisual, works of art, works of applied art, design and architecture, works of folk art and folk art production. [1]

In our research, we focus on the historical library documents, namely in terms of quality digitization depending on their language and font.

2. The Object of the Research

Content libraries are crucially participated and participate in the exchange of information and the building of spiritual values in the past and present. Each national culture seeks to adequately preserve, interpret and make use of written cultural heritage collected by libraries, archives, museums and individuals. A set of written cultural heritage called the national library, which is located in different geographical and institutional space. Creates an archive the most important of information and document values. Their research and knowledge gives us an authentic witness to the history of the nation, economic, political, cultural and social life of various sections of society. Historical library can not be seen only as a set of stored documents, but primarily for the communication of values, their reception, exchange and enrichment. Historical libraries have enormous information wealth, which has the assumption to create digital content of high intellectual quality in the case of using appropriate solutions. Thematic diversity is a specific content for specific groups of users.

We suppose that the language and fonts have the key importance to the effectiveness and the quality of digital content. Qualities of the mentioned objects (language and font) are essential for

old prints primary. In the new documents (after 1918) the font and language shouldn´t pose big problem to create digital content.

In our research we focus on the typology of the font in the historical Slovakia-related printed documents from the Ancient time (16th century) to 1918. The historical Slovakia-related document is a library document, which according to authorship, language, place of publication or the contents relate to Slovakia and the Slovaks, issued or printed in 1918.

3. The Aim of the Research

The primary objective of our research is the basis of outputs from scanners to design optimal rules of digitalization the old Slovakia-related documents to create the highest quality and usable digital content within historical libraries.

For transparency and usability of the knowledge will be created catalog that will set all the fonts that are used in our geographic area for more than four centuries. We assume that some sets of fonts can be found in many printers, because each set progressively often owned several printers. Set of fonts - List of fonts will include all font types, which meet during the research and description of optimal scanner settings (manual and automatic digitization robots) in creating digital content printed using the font types.

4. The Process Solutions

We follow:

1. the basic literature dedicated to the development of fonts and give us space to identify possible fonts that appear in historical Slovakia-related documents:
    a. The history and evolution of our fonts (Húščava, A., 1951),
    b. The history of fonts, books and printer (Mišianik, J., 1956),
    c. The history of font and book (Špetko, J., 1963),
    d. The book's culture and font (Špetko, J., 1969).
2. from the list of printers and printer in Slovakia from 16th to 20th century (see Tab. 1), based on an analysis of historical library documents prepared by Mgr. Eva Augustínová, PhD. [2], which will be the basis for sampling prints used in our study.

4.1. The Research Sample

Based on the list of printers to search the retrospective bibliography[1] available documents and then prepare them digitally sampled. When preparing a digital document image are working on two types of scanners - Treventus and classical handheld scanner Bookeye. As research sample, we identify at least three print produced by a printer, and the top, middle, and end of his tenure. The priority will be the document language, we focus mainly on printing in Slovak language (the basis for language Slovak National Corpus), as well as sample but use the press in other languages, but these samples analyzed we turn our attention to one copy of each language.

1 RIZNER, Ľ.V. *Bibliografia písomníctva slovenského na spôsob slovníka od najstarších čias do konca roku 1900.* Turčiansky Sv. Martin : MS, 1929 – 1934. - MIŠIANIK, J. *Bibliografia písomníctva slovenského na spôsob slovníka od najstarších čias do konca roku XIX. stor*. (Doplnky k Riznerovej bibliografii).Bratislava : SAVU 1946. 300 s. - ORMIS, J.V. *Doplnky a opravy k Riznerovej bibliografii písomníctva slovenského*. Martin : MS, 1935-1952. - ČAPLOVIČ, J. Bibliografia tlačí vydaných na Slovensku do roku 1700. 1. diel. Martin : MS, 1972. 555 s. - ČAPLOVIČ, J. Bibliografia tlačí vydaných na Slovensku do roku 1700. 2. diel. Martin : MS, 1984. 556-1410 s. - FEDOR, M. *Biliografia slovenských kníh 1901-1918.* Martin : MS, 1964. 727 s. - KLIMEKOVÁ, A. a kol. *Bibliografia územne slovacikálnych tlačí 18. storočia.* 6 zväzkov Martin : Slovenská národná knižnica, 2008. – Databáza územne slovacikálnych tlačí 19. storočia. Lokálna databáza v Slovenskej národnej knižnici.

Each of selected documents we scan in three different DPI settings. We explore the differences in the quality of a digital image and its presentation in the digital space in settings 150, 300 and 600 DPI on both scanners.

Created scans will then pass system OCR (Optical Character Recognition), which outputs will serve as the final examination objects.

Optical character recognition is a method that allows us to flip the picture printed characters into text format, which can then edit. For the purposes of our research we use softwere OCR ABBYY FineReader OCR. The software allows us to image the scanned pages of historical documents to create a text document format. This step is necessary for more optimal use of digital content. Scanned page document and will act as a classic text document. For optimal image recognition in the text should use linguistic elements to correctly translate words and phrases. Generation of text from scanned image books can only work with the image quality, because we in our research to generate multiple scans one document at different DPI resolutions and then compare the OCR outputs.

City	**The number of printers**	**City**	**The number of printers**
Banská Bystrica	20	Prešov	11
Banská Štiavnica	17	Púchov	5
Bardejov	13	Revúca	4
Bratislava	88	Rimavská Sobota	8
Bytča	2	Rožňava	6
Čadca	2	Ružomberok	3
Dolný Kubín	4	Senica	3
Holíč	3	Sereď	2
Jasov	2	Skalica	10
Kežmarok	8	Spišské Podhradie	4
Komárno	26	Šamorín, Mliečno	3
Košice	41	Topoľčany	3
Kremnica	2	Trenčín	8
Levice	4	Trnava	19
Levoča	21	Zlaté Moravce	6
Liptovský Mikuláš	4	Zvolen	7
Lučenec	8	Žilina	8
Malacky	4	Komjatice, Košice, Plavecké Podhradie, Rohožník, Šintava	6
Nitra	13		

Tab. 1 The number of printers and printer in Slovakia by cities.

4.2. The Form or the Research

The research results of each research sample will be entered in the form. Any research sample will be assigned to your form, which will contain the name of the printer, year of publication. These data are necessary for subsequent digitization of other historical documents, where on that basis we will be able to determine exactly how devices should be set to optimize digital content creation, other books.

Other data in the form is the document language, font type and size of the document. These data will give us additional information about the document. The most substantial of the data, the language of the document, as it focuses mainly on documents written in Slovak language.

The following data will be recorded separately for Automat Treventus and separately for manual handheld scanner Bookeye. This includes the following data: scan time, DPI values and data from OCR.

Given that we perform complete scans the entire document, but only a few pages, we will indicate the time of scanning a maximum of two pages of a document. From the data thus can derive the approximate time to scan the entire document. This time is obviously indicative, because they scan various complications can occur, which this time is extended.

Scan quality, we examine the values at 150, 300 and 600 DPI. When the individual values we examine differences in quality of generated scan and its readability. We assume that all historical documents will be scanned with 600 dpi to optimize subsequent work with documents.

As a last records on the form will be inserted information from OCR. We will be recorded in detail the effectiveness of OCR without any adjustment. We focus on the unrecognized letters, words and sentences in different types of fonts, data will be reported in percentages. Then we record the duration of the OCR settings to streamline the process and create a higher quality digital content. This time we will indicate in minutes. Recent data for both types of scanners will be made after the effectiveness of the OCR settings. Also this will be referred to a percentage and then we will focus on different percentages. Our goal is to make the settings, the percentage of unrecognized text much lower.

Statistical evaluation of outcomes and their analysis, the optimal settings will be the foundation to create sampler fonts and database with the necessary data for streamline digitization of historical and not only the Slovakia-related documents.

5. Conclusion

With digitalization historical documents we must take account of the type of font and of the language of document. The result of our research will be The book of patterns type of fonts used in the Slovakia historical printed documents that will serve as a basis for the program to flip the image quality of printed characters into text format. Become a support for effective digitalization of written heritage of Slovakia.

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Accessibility in Digitisation

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Abstract. Our article looks at the process which enables digital access to the scientific and cultural heritage in public and university. The aim of this paper is to investigate the standards of digitisation and the projects of digitisation and accessibility in Europe and the Slovak Republic. Digitisation is an important means for ensuring greater access to usage of cultural material. Key role in accessibility play international standards for digitization and especially for accessibility of digital content. The subject of cultural and scientific heritage is in the center of interest and user is center of interest of the process of accessing and presenting objects. We are still forgotten that the purpose of digitisation is also distribution of digitized material. Digitisation is just a tool, the one of the goals is accessibility. University environment offers a variety of options for making content accessible. By supporting accessibility the system can be available to simplify the way of distribution and archiving of scientific and learning materials.

Keywords: Digitisation. Accessibility. Standards for Digitisation. Cultural and Scientific Heritage.

1. Introduction

Digitisation of analogue content is of great significance in the library and information science. Accessibility of cultural materials does not receive enough attention. The presentation layer cannot be effective without creating a direct link to the monitored user`s behaviour. The subject of cultural and scientific heritage is in the center of interest and user is the center of interest of process of accessing and presenting objects.

Research problem of accessibility of digital content is much more complex. The aim of digitization is not only accessibility, but also preservation and protection of analogue materials. Accessibility is only part of complex work flow of digitization (selection, classification, logistic, conservation, scanning, post-processing, metadata, infrastructure, hardware, software, long term preservation and archiving etc...). Only part of digitized materials is possible exposure for general public according copyright law. Key role in accessibility play international standards for digitization and especially for accessibility of digital content.

2. International Standards for Digitisation (Especially for Accessibility of Digital Content)

Digitisation is conversion of analogue signal to digital signal. It can be representation of images, sounds, 3D objects, documents, videos... The result of digitisation is digital representation – digital form of the object. Digitisation allows:

- keeping a digital image of the document/object in digital form in a digital repozite
- computer processing of digital material
- transferring and accessing digital image/object over the network, for example

The whole process of digitisation must comply with international standards.

image type	format
Text	HTML 4.01 HyperText Markup Language [1], XHTML 1.0 The Extensible HyperText Markup Language [2]

	Information technology - Open Document Format for Office Applications (OpenDocument) v1.0 [3]
	Portable Document Format (PDF) [4]
photographic images	JPEG/SPIFF [5]
Graphic non-vector images	GIF/PNG [5]
vector images	SVG [5]
video (downloading)	MPEG-1/AVI/WMW/Apple Quicktime formats [5]
video (streaming)	ASF/WMV/Apple Quicktime formats [5]
audio (downloading)	MP3/RA/WMA [5]
video (streaming)	MP3/RA/WMA [5]
3D object	Web3D Consortium [6], X3D [7], QuickTime VR[8]

Tab. 1. Standards for digitalised objects.

3. Digitisation in Europe and the Slovak republic

In ancient times, the Library of Alexandria held a collection representing up to 70% of the whole human knowledge. The challenge of these days of the digital age is to achieve something similar but with the results preserved. The initiative of libraries created space for the digitisation of various contents (books, journals, films, maps, photographs, music, etc...), accessible to all and preserved for future generations. Digitisation is an important means to ensure wider access and usability to cultural material. Coordinated cooperation would lead the member states to digitising their cultural heritage with greater coherence in the selection of material and would reduce the risk of duplicate digitising and avoid the need to re-digitise.

3.1. European Digitisation Projects

Digital Agenda for Europe

In March 2011 the European Commission published the European 2020 Strategy. This strategy is responded to the economic crisis of recent years and is prepared for the European economy challenges of the next decade. The **Digital Agenda for Europe** is particularly important initiative for community of information, repository and fund institutions. [9]

Since the implementation of this agenda are expected incentives for innovation, economic growth, development and quality of life business. A particular benefit of such agenda to be better health care, better transport solutions, cleaner environment, new media opportunities and easier access to public services and the new, universal access to cultural content. The digital libraries initiative is part of the Commission's Digital Agenda for Europe. [9]

Enumerate Project

The **ENUMERATE** Project was started in the UK. Its main objective is to provide reliable statistics on digitisation, digital preservation and online access to cultural heritage in Europe. The EC-funded ENUMERATE network conducted its first ENUMERATE Core Survey. The aim of this survey was to develop a clear understanding of the progress of digitisation in European cultural heritage sector. The overall response from all countries is quite satisfactory. Number of institutions involved in the project is large enough to provide reliable data on the level, type and size of the institution. [10]

MinervaEC

MinervaEC is a Thematic Network in the area of cultural, scientific information and scholarly content. Its goal is to facilitate the creation of added value products and services at European level,

to improve awareness of the state-of-the-art in the sector, to contribute to the overcome of fragmentation and duplication of digitisation activities of cultural and scientific content and to maximise cooperation among the Member States. [11]

NUMERIC

The Numeric Study aimed to measure the progress of the digitisation of Europe's cultural heritage.One of Europe's key Information Society policy objectives is to make the content and digitally preserved materials of archives, museums and libraries more widely available. [12]

Other project

Other projects dealing with the programs for research and technological development in digitization and digital cultural and scientific content are CALIMERA, ERPANET, TEL-ME-MOR, DELOS, BRICKS, PRESTOSPACE, etc.

3.2. Digitisation in Slovak republic

Slovak Digital Library

In social discourse the efforts of the Slovak National Library (SNL) resonate aimed at creating the Slovak Digital Library, which is related to the process of digitising the national archives and library collections. The aim of the national Digital Library Project is to digitise and make available the written cultural and scientific heritage accessible to all citizens of Slovakia, Europe and the world. The specific objective of the project is to digitise all the Slovak and Slovakia-related documents (books, scientific journals, anthologies, newspapers and other print, old and rare prints, archival and other documents). [13]

4. Digitisation in the Context of Accessibility

In the library and information science, digitisation is seen as a process consisting of the activities ranging from selection, through conversion, description, storage, preservation, distribution and provision of services. Online accessibility of cultural materials will make it possible for citizens throughout Europe to access and use it for leisure, studies or work.

Digitising materials is of irreplaceable importance for linguistic research. Digitisation creates a unique and huge massif of digital texts, which will receive a thorough and complete understanding of the development of Slovak language (the national corpus of the Slovak language, lexicography, etymology). The digital library will also be an indispensable source of knowledge and information for historical research, the creation of encyclopedias, dictionaries, interpretive, scientific and technical synthetic works.

The Memory of Slovakia Project

The Memory of Slovakia - National Centre of Excellence in research, Preservation and Accessibility of the Cultural and Scientific Heritage top includes a digitisation centre located in the University Library of the University of Žilina. The center is equipped with cutting-edge technology which makes it one of its kind in Slovakia to keep the pace with other European centres of excellence. The strategic objective of the project is to establish the national centre of excellence in research, preservation, conservation and accessibility of cultural and scientific heritage with internationally recognized basic research. The project will improve the technical infrastructure of top research institutions in the protection and accessibility of cultural and scientific heritage in the region of Žilina, as the current level of technical infrastructure does not implement a number of research activities at the appropriate level of quality, or it does not allow them to implement research at all. The project significantly improves the conditions of the

educational process and the preparation of a new generation of scientists. The specific objective is to achieve significant results in research, conservation and presentation of cultural and scientific heritage. [14]

5. Conclusion

Digital content is necessary, especially in the university environment. We can see a plenty of opportunities and advantages resulting from digitisation of materials. The system to ensure accessibility is important because of good access to databases of cultural heritage. We strive to investigate the best system for accessing cultural and scientific heritage.

The Memory of Slovakia will improve the technical infrastructure of top research institutions in the protection and accessibility of cultural and scientific heritage. All documents have to be available on the web and it would be possible to search them.

The university environment offers a variety of options for accessing content. By creation of some system of accessibility we can establish a simplified way for distribution and archiving of scientific and study materials. Students can learn and easily navigate the necessary resources and will always have access to the current versions of content. For added benefit we can consider the savings on the costs of printing and publication.

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